

Delay-Varying Repetitive Control with Application to a Walking Piezo Actuator

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Abstract

The performance of systems that exhibit repetitive disturbances can be significantly improved using repetitive control. If the repetitive disturbance is periodic with respect to time, perfect asymptotic disturbance rejection can be achieved by well known methods. However, many systems have a repetitive nature with respect to a variable other than time. For this type of systems, we propose a delay-varying repetitive control (DVRC) method, which employs a time-varying delay in the repetitive controller that is continuously adjusted based on the repetitive variable. An \mathcal{H}_∞ norm-based criterion is derived that guarantees stability of the time-varying delay system for a given range of variations of the repetitive delay. The strengths of this new repetitive control scheme are shown by applying it to a nano-motion stage driven by a walking piezo actuator.

Key words: Repetitive control; time-delay systems; nano-motion; piezo actuators.

1 Introduction

The performance of systems that perform repetitive tasks or that are subject to repetitive disturbances can be improved significantly using repetitive control (RC). In most available RC methods it is assumed that the disturbances are periodic with respect to time. This leads to a fixed value for the repetitive delay in the memory loop of RC, for which guaranteed properties can be obtained [5,7,8]. However, many systems have a repetitive nature with respect to another variable than time. Existing RC schemes with a constant repetitive delay are not applicable in these circumstances.

Several solutions for the application of RC to systems that are subject to repetitive disturbances with a slowly varying period with respect to time have already been proposed in literature [1,2]. In contrast with [1,2], the adaptive RC scheme proposed in [6] does not change the sampling frequency, but adapts the delay in the memory loop based on a physical model of the time-varying character of the repetitive delay. Since the variation is

assumed to be slow in time, the delay is adjusted at a fixed rate that is much less than the controller sampling rate. Unfortunately, no stability guarantees are given for these cases. In addition, the assumption on the slow variation of the period is not valid in various applications, including the walking piezo actuator in this paper.

An alternative method is high-order RC, which uses multiple memory loops to provide robustness against small variations in the period-time of repetitive disturbances [3,12,13]. Another line of research considers systems that exhibit spatially repetitive disturbances, e.g., disturbances that are periodic with respect to a rotation angle in motor/gear transmission systems [4] and internal combustion engines [15]. Transformation of these systems to the rotational-angle domain renders the delay constant in the new independent variable being the rotation angle. However, the design of stabilizing feedback controllers becomes very complicated since the transformed systems are nonlinear.

In this paper, we propose a novel method called delay-varying repetitive control (DVRC) for systems that have a repetitive variable other than time. DVRC makes use of a measured or observed repetitive variable, e.g., the angular orientation of the legs in the walking piezo actuator, to adjust the repetitive delay in the RC scheme. The proposed method overcomes many of the mentioned

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drawbacks of existing schemes, e.g., it is applicable in real-time at a fixed sampling-time and it can cope with fast and large variations in the repetitive delay. As the resulting closed-loop system is time-varying in nature, a formal stability analysis is required. A stability proof of DVRC is given incorporating time-varying delays, leading to frequency domain design criteria for the learning filters. Note that although design methods for robust RC are available [10,17–19], robustness to varying delays has not been considered in the RC literature. The proposed DVRC method is applied to a walking piezo actuator, used to drive a nano-motion stage, which show the significant improvement of DVRC compared to standard RC.

2 Repetitive control

RC is applied to control loops in which repetitive disturbances and/or references are present. The repetitive nature of the disturbances (and references) means that these disturbances are periodic with respect to some variable α in the system. In standard RC schemes [5,7,8] this repetitive variable α is the (continuous) time t , meaning that the repetitive disturbances d_r are periodic with respect to time, i.e., $d_r(t+P_\alpha) = d_r(t)$ for all $t \in \mathbb{R}_+$ and some $P_\alpha \in \mathbb{R}_+$, called the repetitive period. In a discrete-time implementation one normally chooses the sampling time T_s of the controller such that $P_\alpha = T_s N$, with $N \in \mathbb{N}$ the number of samples corresponding to the repetitive period. To suppress the periodic disturbances in time, a memory loop is included in the discrete-time repetitive controller using a constant delay of N samples.

To explain standard RC, in which the repetitive variable α is equal to time, consider the schematic representation of a feedback controlled SISO system with RC as shown in Fig. 1, where $G(z)$ denotes the transfer function of a linear time-invariant discrete-time system with input u and output y . The feedback controller is denoted by $K(z)$. The tracking error is given by $e = r - y$, where r is the reference. The repetitive controller $M(z, \alpha)$ is depicted within the dashed block, in which $L(z)$ is the learning filter with a delay of l_d samples and $Q(z)$ is the linear-phase robustness filter with a time delay of q_d samples. Since in standard RC the repetitive variable α is time, the repetitive delay, denoted in Fig. 1 by $z^{-N(\alpha)}$, is constant, i.e., $N(\alpha) = N = P_\alpha/T_s$ (samples).

For standard RC with a constant repetitive delay N , the transfer function of the repetitive controller $M(z, \alpha) = M(z)$, i.e., the transfer function between the tracking error e and the output w , equals

$$M(z) = \frac{W(z)}{E(z)} = \frac{L(z)Q(z)z^{-(N-l_d-q_d)}}{1 - Q(z)z^{-(N-q_d)}}, \quad (1)$$

where $W(z)$ and $E(z)$ are the z -transforms of the time signals w and e , respectively. The sensitivity function

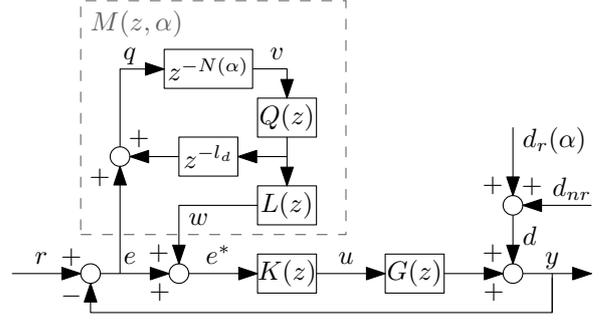


Fig. 1. Block diagram of a DVRC setup.

$S(z)$, relating the disturbances d to the tracking error e is given by

$$S(z) = \frac{E(z)}{D(z)} = \frac{1}{1 + G(z)K(z)(1 + M(z))}. \quad (2)$$

Substitution of (1) in (2) gives $S(z) = \bar{S}(z)M_s(z)$, where $\bar{S}(z) = (1 + G(z)K(z))^{-1}$. The modifying sensitivity function $M_s(z)$ [3] is given by

$$M_s(z) = \frac{1 - Q(z)z^{-(N-q_d)}}{1 - Q(z)z^{-(N-q_d)}(1 - \bar{T}(z)L(z)z^{+l_d})}, \quad (3)$$

where $\bar{T}(z) = G(z)K(z)/(1 + G(z)K(z))$ is the complementary sensitivity function without RC.

2.1 Stability when the repetitive variable is time

For a constant delay of N samples, the stability of the system in Fig. 1 is guaranteed if the following two conditions are fulfilled [14]:

- (1) the sensitivity $\bar{S}(z)$ has all poles in the open unit circle of the complex plane, and
- (2) for all $z \in \mathbb{C}$ with $|z| = 1$

$$|Q(z)(1 - \bar{T}(z)L(z)z^{+l_d})| < 1. \quad (4)$$

These conditions follow from small gain arguments by considering Fig. 1 as the feedback interconnection of $H(z) = Q(z)z^{-l_d}(1 - \bar{T}(z)L(z)z^{+l_d})$, being the transfer function from v to q , and a constant delay block z^{-N} , for which $|z^{-N}| = 1$ for all $z \in \mathbb{C}$ with $|z| = 1$.

2.2 Filter design when the repetitive variable is time

From the criterion (4) it follows that a straightforward choice for the learning filter is the inverse of the complementary sensitivity function, i.e., $L(z) = \bar{T}^{-1}(z)$. In case an exact proper and stable inverse cannot be obtained, e.g., when $\bar{T}(z)$ is non-minimum phase and/or non-proper, an approximation of the inverse is

made, e.g., using the zero-phase-error-tracking-control (ZPETC) method [16].

For the determination of the fixed delay value N , the tracking error e containing the repetitive disturbances d_r is measured without RC. From the spectrum of e , the repetitive period P_α can be determined as the lowest harmonic in the signal. The fixed delay value then follows as $N = P_\alpha/T_s$, as discussed before.

The Q -filter is designed to account for mismatches between $L(z)$ and $T^{-1}(z)$. For standard RC with a fixed delay, the Q -filter is designed such that the criterion (4) is fulfilled. The Q -filter is constructed to have a linear phase of q_d samples, which are compensated by removing q_d samples from the memory loop. The filtering with the Q -filter will then effectively have a zero-phase [14]. The introduced time delay of the L and Q -filters can be compensated for in the memory loop of N samples (see Fig. 1) by reducing the delay to $N - q_d - l_d$ samples instead of N . In this way the total delay in the memory loop of RC is equal to N samples, as desired.

At low frequencies the performance of the DVRC scheme is determined by how close $L(z)$ resembles $T(z)^{-1}$. The design of the Q -filter, required in order to meet the criterion (4), determines the frequency up to which the learning scheme is effective.

3 Delay-varying repetitive control

3.1 Problem formulation

In many practical situations disturbances are periodic with respect to other variables α than time, e.g., angles in rotating systems or the angular orientation of the piezo legs in the walking piezo actuator of Section 5. The only properties that we impose on the repetitive variable α is that it is strictly increasing in time¹ and that the relevant disturbances $d_r(\alpha)$ are periodic in α : there is a $P_\alpha \in \mathbb{R}_+$ called the repetitive period such that $d_r(\alpha + P_\alpha) = d_r(\alpha)$ for all $\alpha \in \mathbb{R}_+$. Clearly, variations in the rate $\dot{\alpha}$ result in disturbances that are not fully repetitive in time. To suppress these types of disturbances, we propose an alternative RC scheme, referred to as delay-varying repetitive control (DVRC). The rate-variation of the repetitive variable α is incorporated in the scheme by making the repetitive delay time-varying as $N(\alpha(t))$.

The assumption that the repetitive variable α is strictly increasing in time and $\alpha(0) = 0$ ² guarantees that there is a one-to-one correspondence between the repetitive

¹ In case α is strictly decreasing one can take $-\alpha$ as the repetitive variable.

² In case $\alpha(0) = a \neq 0$ the same reasoning applies by replacing α by $\tilde{\alpha}$ with $\tilde{\alpha}(t) = \alpha(t) - a$, $t \in \mathbb{R}_+$.

variable $\alpha \in \mathbb{R}_+$ and the (continuous) time $t \in \mathbb{R}_+$. Hence, for each value of $\alpha(t)$ there is a unique corresponding time $t = \alpha^{-1}(\alpha(t))$, where $\alpha^{-1} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ denotes the inverse function of α . Clearly, $t = \alpha^{-1}(\theta) \in \mathbb{R}_+$ is the time at which the repetitive variable α attains the value $\theta \in \mathbb{R}_+$. The time-varying delay $N(\alpha(t)) \in \mathbb{R}_+$ in $z^{-N(\alpha(t))}$ at time $t \in \mathbb{R}_+$ is equal to

$$N(\alpha(t)) = t - \alpha^{-1}(\alpha(t) - P_\alpha) \text{ for } \alpha(t) \geq P_\alpha \quad (5)$$

in continuous time. The calculated delay $N(\alpha(t))$ is the elapsed time between the current time t (at which the repetitive variable is equal to $\alpha(t)$) and the time at which the repetitive variable α was exactly one repetitive period P_α less than $\alpha(t)$.

In a discrete-time implementation with sampling time $T_s > 0$ as used here, all signals including the repetitive variable α are considered at discrete times kT_s , $k \in \mathbb{N}$. To accommodate for this discrete nature in (5), we determine at each sample k the sample index $k^* \in \mathbb{N}$ at which α is closest to $\alpha(kT_s) - P_\alpha$, which is given by

$$k^*(\alpha(kT_s)) = \arg \min_{l \in \mathbb{N}} (\alpha(lT_s) - \alpha(kT_s) + P_\alpha)^2. \quad (6)$$

The time-varying delay as in (5) can now be approximated as

$$N(\alpha_k) = k - k^*(\alpha_k) \text{ for } \alpha_k \geq P_\alpha, \quad (7)$$

where $\alpha_k = \alpha(kT_s)$.

3.2 Design procedure for DVRC

To design the DVRC scheme the following procedure can be used.

- (1) Determine the repetitive variable α and the repetitive period P_α . The repetitive delay $N(\alpha_k)$ is online determined as in (7), which results in the implementation of the time-varying delay $z^{-N(\alpha_k)}$ at $k \in \mathbb{N}$.
- (2) The complementary sensitivity $\bar{T}(z)$ is not effected by the time-varying delay $z^{-N(\alpha_k)}$. The learning filter $L(z)$ for DVRC can therefore be designed analogous to standard RC in such a way that $L(z)$ is close to $\bar{T}^{-1}(z)$.
- (3) Let the time-varying delay $N(\alpha)$ satisfy $N(\alpha_k) \in [m, M] \cap \mathbb{N}$, for $k \in \mathbb{N}$, where $m \in \mathbb{N}$ and $M \in \mathbb{N}$ denote the minimum and maximum repetitive delay, respectively. To guarantee stability of the DVRC scheme with time-varying delay $N(\alpha)$, the linear-phase Q -filter is designed to fulfill the following:
 - (a) $\bar{S}(z)$ has all poles in the open unit circle of the complex plane, and

(b) for all $z \in \mathbb{C}$ with $|z| = 1$

$$|Q(z) (I - \bar{T}(z)L(z)z^{+l_d})| < \frac{1}{\sqrt{M - m + 1}}. \quad (8)$$

Note that criterion (4) is not valid anymore to guarantee stability of DVRC due to the time-varying delay. The sufficiency of (8) for stability is proven next.

4 Stability analysis

In this section, it is proven how (8) is related to guaranteeing stability of the RC scheme when the repetitive delay lies in a given range, i.e., $N_k := N(\alpha_k) \in [m, M] \cap \mathbb{N}$, where $m, M \in \mathbb{N}$ with $0 \leq m \leq M$. If we ignore the external signals d and r for the moment, the system in Fig. 1 can be represented as the feedback interconnection of the discrete-time system

$$x_{k+1} = Ax_k + Bv_k; \quad q_k = Cx_k \quad (9a)$$

and the varying delay block

$$v_k = q_{k-N_k}, \quad (9b)$$

where $x_k \in \mathbb{R}^{n_x}$ is the state and $v_k \in \mathbb{R}^{n_v}$ and $q_k \in \mathbb{R}^{n_q}$ with $n_v = n_q$ are the interconnection variables at discrete time $k \in \mathbb{N}$. System (9a) is a state space representation of the transfer function $H(z) = Q(z)z^{-l_d} (I - \bar{T}(z)L(z)z^{+l_d})$ between v and q in Fig. 1. Hence, Fig. 1 (with $r = d = 0$) reduces to Fig. 2 using this perspective.

The varying delay block (9b) can also be written in state space notation as

$$\zeta_{k+1} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ I_{n_q} & 0 & 0 & \dots & 0 & 0 \\ 0 & I_{n_q} & 0 & \dots & 0 & 0 \\ \vdots & & & \ddots & \vdots & \\ 0 & 0 & \dots & 0 & I_{n_q} & 0 \end{bmatrix} \zeta_k + \begin{bmatrix} I_{n_q} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} q_k, \quad (10a)$$

$$v_k = \begin{bmatrix} \Gamma_1(N_k) & \dots & \Gamma_M(N_k) \end{bmatrix} \zeta_k + \Gamma_0(N_k)q_k \quad (10b)$$

with $\zeta_k = (q_{k-1}^T, \dots, q_{k-M}^T)^T$ and for $i = 0, 1, \dots, M$ The matrix $\Gamma_i(N) = I_{n_q}$ when $N = i$, and $\Gamma_i(N) = 0$ when $N \neq i$. Here, I_m denotes the identity matrix of dimension $m \times m$. Although in the setup in Fig. 1 all signals are scalar valued (i.e., $n_q = n_v = 1$), we present the stability for MIMO plants for reasons of generality.

Definition 1 [ℓ_2 gain] *A discrete-time system*

$$x_{k+1} = f(x_k, N_k, v_k); \quad q_k = g(x_k, N_k, v_k), \quad (11)$$

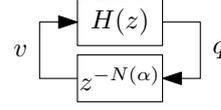


Fig. 2. Feedback interconnection of a system $H(z)$ with a time-varying delay $z^{-N(\alpha)}$.

with state x_k , (disturbance) input v_k , parametric uncertainties N_k and output q_k at discrete time $k \in \mathbb{N}$ is said to have a (robust) ℓ_2 (induced) gain of γ for uncertainties in Υ , if γ is the minimal (or infimal) value of $\tilde{\gamma}$ satisfying for any input sequence $\{v_k\}_{k \in \mathbb{N}}$ with $\sum_{k=0}^{\infty} \|v_k\|^2 < \infty$ and any sequence $\{N_k\}_{k \in \mathbb{N}}$ of uncertainties with $N_k \in \Upsilon$, $k \in \mathbb{N}$, the inequality

$$\sum_{k=0}^{\infty} \|q_k\|^2 \leq \tilde{\gamma}^2 \sum_{k=0}^{\infty} \|v_k\|^2,$$

where $\{q_k\}_{k \in \mathbb{N}}$ is the corresponding output sequence with initial condition $x_0 = 0$.

Theorem 2 *The following statements are equivalent:*

- (1) *The linear system (9a) has ℓ_2 gain smaller than γ .*
- (2) *The \mathcal{H}_∞ norm $\|H(z)\|_\infty := \sup_{z \in \mathbb{C}, |z|=1} \bar{\sigma}(H(z))$ with $H(z) = C(zI - A)^{-1}B$ is smaller than γ , where $\bar{\sigma}$ denotes the maximum singular value.*
- (3) *There exist a matrix P and a $\beta \geq \frac{1}{\gamma}$ satisfying*

$$\begin{bmatrix} P - A^T P A - \beta^2 C^T C & -A^T P B \\ -B^T P A & I - B^T P B \end{bmatrix} \succeq 0 \text{ and } P \succ 0. \quad (12)$$

Now we provide a (tight) upper bound on the ℓ_2 gain of the time-varying delay system (10).

Theorem 3 *Consider the delay system (9b) given by $v_k = q_{k-N_k}$ that can be represented in a state space realization as in (10). Let the varying N_k , $k \in \mathbb{N}$ be contained in $[m, M] \cap \mathbb{N}$ with $m, M \in \mathbb{N}$ and $m \leq M$. The ℓ_2 gain of the delay system (10) with disturbance set $[m, M] \cap \mathbb{N}$ is equal to $\sqrt{M - m + 1}$.*

Proof: We will prove that the system (10) has

$$W(\zeta_k) := \sum_{i=m+1}^M (M-i+1) \|q_{k-i}\|^2 + \sum_{i=1}^m (M-m+1) \|q_{k-i}\|^2$$

as a storage function for the supply rate $s(q_k, v_k) = (M - m + 1) \|q_k\|^2 - \|v_k\|^2$, i.e.

$$W(\zeta_{k+1}) - W(\zeta_k) \leq (M - m + 1) \|q_k\|^2 - \|v_k\|^2 \quad (13)$$

for all $k \in \mathbb{N}$. By standard arguments, this implies that the ℓ_2 gain of the delay system (10) with disturbance set $[m, M] \cap \mathbb{N}$ is smaller than or equal to $\sqrt{M - m + 1}$.

To prove (13), consider

$$\begin{aligned}
W(\zeta_{k+1}) - W(\zeta_k) &= \\
&\sum_{i=m+1}^M (M-i+1) \|q_{k+1-i}\|^2 + \sum_{i=1}^m (M-m+1) \|q_{k+1-i}\|^2 \\
&- \sum_{j=m+1}^M (M-j+1) \|q_{k-j}\|^2 - \sum_{j=1}^m (M-m+1) \|q_{k-j}\|^2 \\
&\stackrel{l:=i-1}{=} \sum_{l=m}^{M-1} (M-l) \|q_{k-l}\|^2 + \sum_{l=0}^{m-1} (M-m+1) \|q_{k-l}\|^2 \\
&- \sum_{j=m+1}^M (M-j+1) \|q_{k-j}\|^2 - \sum_{j=1}^m (M-m+1) \|q_{k-j}\|^2 \\
&= (M-m+1) \|q_k\|^2 - \sum_{l=m}^M \|q_{k-l}\|^2 \\
&\leq (M-m+1) \|q_k\|^2 - \|v_k\|^2,
\end{aligned}$$

where in the last inequality we used that $v_k = q_{k-N_k}$ for some $N_k \in \{m, m+1, \dots, M\}$ (see (9b)). This shows that the ℓ_2 gain is smaller than or equal to $\sqrt{M-m+1}$. In [11] it is shown that the ℓ_2 gain of the time-varying delay system (10) is larger than or equal to $\sqrt{M-m+1}$. Hence, the ℓ_2 gain is equal to $\sqrt{M-m+1}$, thereby completing the proof. ■

Based on Theorem 3, we can prove the following stability result for the closed-loop system (9) including an explicit construction of a Lyapunov function.

Theorem 4 Consider system (9a) with A Schur and ℓ_2 gain strictly smaller than $\frac{1}{\sqrt{M-m+1}}$ for $m, M \in \mathbb{N}$ and $m \leq M$. Then the system (9) with time-varying $N_k \in [m, M] \cap \mathbb{N}$, $k \in \mathbb{N}$ is globally asymptotically stable.

Proof: Take the Lyapunov function candidate $V(\xi_k) = \bar{V}(x_k) + W(\zeta_k)$ with $\bar{V}(x_k) = x_k^T P x_k$ and P satisfying (12) for some $\beta^2 > M - m + 1$ and $W(\zeta_k)$ as in the proof of Theorem 3. Hence, using the inequality in the proof of Theorem 3 and the fact that due to (12) we have that $\bar{V}(x_{k+1}) - \bar{V}(x_k) \leq -\beta^2 \|x_k\|^2 + \|v_k\|^2$ for all $k \in \mathbb{N}$, we obtain for all $k \in \mathbb{N}$

$$V(\xi_{k+1}) - V(\xi_k) \leq (M - m + 1) \|q_k\|^2 - \beta^2 \|x_k\|^2.$$

Since $\beta^2 > M - m + 1$, this gives

$$V(\xi_{k+1}) - V(\xi_k) \leq -(\beta^2 - (M - m + 1)) \|x_k\|^2, \quad (14)$$

which directly proves Lyapunov stability of the closed-loop system (9). Indeed, (14) proves Lyapunov stability as $V(\xi_{k+1}) \leq V(\xi_k)$ for all $k \in \mathbb{N}$ and $c_1 \|\xi\|^2 \leq V(\xi) \leq c_2 \|\xi\|^2$ for all ξ for some $0 < c_1 \leq c_2$. To show that $\lim_{k \rightarrow \infty} \xi_k = 0$, note that by summing (14)

for $k = 0, 1, \dots, \ell$ we obtain that $V(\xi_{\ell+1}) - V(\xi_0) \leq -\alpha \sum_{k=0}^{\ell} \|q_k\|^2$ with $\alpha := \beta^2 - M + m - 1 > 0$ and thus $\sum_{k=0}^{\infty} \|q_k\|^2 \leq \frac{1}{\alpha} V(\xi_0)$. This implies that $q_k \rightarrow 0$ ($k \rightarrow \infty$) and due to (9b) also that $v_k \rightarrow 0$ ($k \rightarrow \infty$). Since A is Schur, this yields that $\lim_{k \rightarrow \infty} x_k = 0$ and thus $\lim_{k \rightarrow \infty} \xi_k = 0$. ■

The above result shows that the size of the *variation* in the delay determines the requirement on the \mathcal{H}_∞ norm (ℓ_2 gain) of the linear system, not the (absolute) size of the delay itself. Actually in case there is no variation in the delay (so $m = M$) it suffices for closed-loop stability to have A Schur and a \mathcal{H}_∞ norm $\|H(z)\|_\infty$ strictly smaller than 1, which recovers the original conditions (4) for the standard RC scheme with constant repetitive delay. The \mathcal{H}_∞ norm conditions become more stringent if the delay is time-varying. In a similar manner as above it can also be shown that under the hypotheses of Theorem 4 the closed-loop system (9) is bounded-input bounded-output (BIBO) stable and input-to-state stable (ISS) when external inputs are present (e.g., the references r and d as in Fig. 1), see [11] for more details. The Lyapunov function constructed in the proof of Theorem 4 plays an important role in this analysis.

Remark 5 Alternative frequency domain characterizations for stability of discrete-time delay systems as in (9) are given in [9], although not in a form (8). In addition, these characterizations are in certain situations more conservative than our \mathcal{H}_∞ based conditions [11].

5 Application to nano-motion stage

5.1 Nano-motion stage

The nano-motion stage (Fig. 3) is driven by a walking piezo motor, which consists of four bimorph piezoelectric drive legs. The drive pads of the legs are pressed against the drive strip of a one degree-of-freedom (DOF) stage using a motor suspension and preload springs such that the (x_m, y_m, z_m) -axes of the motor coincide with the (x, y, z) -axes of the stage. The position of the stage is measured using an optical incremental encoder with a resolution of 0.64 nm. The movement of the back of the motor housing in y_m -direction is measured using a capacitive sensor with a resolution of 0.44 nm.

The drive legs of the walking piezo motor employ a bimorph principle (Fig. 4) through two electrically separated piezo stacks. In Fig. 4 it can be seen that the piezo legs are driven by four independent waveforms $V_i(t)$ (V), $i \in \{1, 2, 3, 4\}$. Each pair of piezo legs, $p_1 = \{A, D\}$ and $p_2 = \{B, C\}$, is driven by two waveforms. For more details see [11].

The repetitive variable α for the walking piezo actuator is the angular orientation of the legs on the tip trajec-

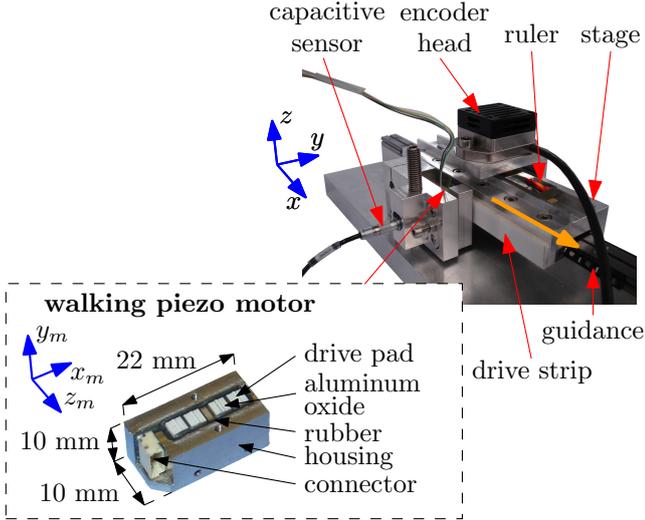


Fig. 3. Nano-motion stage with the walking piezo actuator.

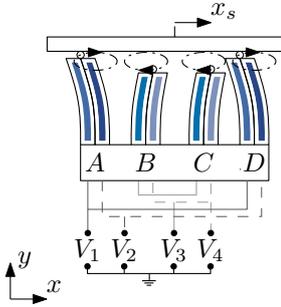


Fig. 4. Working principle of the walking piezo motor with leg trajectories for sinusoidal waveforms $V_i(t)$, $i \in \{1, 2, 3, 4\}$.

tory. The repetitive period equals $P_\alpha = 2\pi$ rad, i.e., one complete cycle of the piezo legs.

5.2 Control configuration

The shape of the tip trajectories of the legs is fixed and described by the electric waveforms to the piezo motor, with input the angular orientation of the drive legs $\alpha(t)$. For feedback control, the angular frequency $\dot{\alpha}(t) = 2\pi f_\alpha(t)$ of the legs f_α (Hz) is chosen as the control input to the system, i.e., $u(t) = f_\alpha(t)$ in Fig. 1. The output of the system is the stage position $x_s(t)$. Using a measured frequency response function (FRF) from the angular frequency $f_\alpha(t)$ (Hz) to the stage position $x_s(t)$ (nm), the feedback controller and the learning filters are designed. First, a parametric model containing a pure integrator, two resonances and one anti-resonance is fitted to the measured FRF as $\hat{G}(s) = \frac{2\pi}{s} \frac{c}{s^2 + 2\pi f_{p1} b_{p1} s + (2\pi f_{p1})^2} \frac{s^2 + 2\pi f_{z1} b_{z1} s + (2\pi f_{z1})^2}{s^2 + 2\pi f_{p2} b_{p2} s + (2\pi f_{p2})^2}$, where $c = 14.9 \cdot 10^9$, $f_{p1} = 527$ Hz, $b_{p1} = 0.033$, $f_{z1} = 624$ Hz, $b_{z1} = 0.02$, $f_{p2} = 650$ Hz and $b_{p2} = 0.175$. The measured system FRF shows a time delay of 0.75 ms, which is caused by calculational, sampling and quantization effects. The delay is incorporated in the

model \hat{G} after discretization by multiplying the discrete model by a discrete-time delay z^{-3} .

The system of Fig. 3 has an inherent nonlinearity since the output $x_s(t)$ contains for a constant input drive frequency $f_\alpha(t)$ repetitive components with other periods than $1/f_\alpha$ (s). This nonlinearity is caused by the harmonic components in the waveform generation [11], resulting in a repetitive movement of the drive legs. The disturbances introduced by the walking movement are fully repetitive with respect to the angular orientation α . The system is considered to be composed of a linearized system model $G_{lin}(z) = X_s(z)/F_\alpha(z)$, which is used for the feedback control, and a nonlinear disturbance generating model, which generates the repetitive disturbance $d_r(\alpha) = g_{nlin}(\alpha)$ (see [11] for more details).

A continuous-time controller $K(s)$ is designed using loopshaping techniques as $K(s) = k \frac{s+2\pi f_{zc}}{s}$, where the gain $k = 2.8 \cdot 10^{-3}$ and the $f_{zc} = 5$ Hz, resulting in a closed-loop bandwidth $f_{BW} = 5$ Hz. The controller is then discretized using a Tustin discretization at a sampling frequency of $f_s = 4$ kHz.

5.3 Learning filters DVRC

The tracking error for an experiment with a reference velocity $\dot{r} = 10 \mu\text{m/s}$ shows on the first sight a repetitive structure. The power spectral density (PSD) of a part of the repetitive error shows that on average over a larger time span a base repetitive frequency of 1.98 Hz is present, which corresponds to $N = 2020$ samples for a sampling frequency of $f_s = 4$ kHz. However, a closer look shows that the period-time of the repetitive disturbances is not constant over time as can be seen in Fig. 5. The repetitive delay $N(\alpha)$ shows for $t > 40$ s, i.e., after the transient response, a fast variation in the range $N(\alpha) \in [2006, 2029]$ samples. The amount of variation, i.e., $M - m$ in Section 4, is a function of the reference velocity. Therefore, the Q -filter should be designed for the worst-case range of variation in $N(\alpha)$ over all relevant references. For the working range of the nano-motion stage of Fig. 3 with velocities ranging from nanometers per second to millimeters per second the worst case variation in repetitive delay equals $M - m = 180$ samples.

The learning filter $L(z)$ is derived as a proper, stable approximation of the inverse of the complementary sensitivity function $\hat{T}(z) = \hat{G}(z)K(z)/(1 + \hat{G}(z)K(z))$ using the ZPETC method [16], i.e., $L(z)\hat{T}(z) \approx 1$.

For a variation in the repetitive delay of $M - m = 180$ samples, the \mathcal{H}_∞ norm bound in the stability criterion (8) equals $\frac{1}{\sqrt{M-m+1}} = 1/\sqrt{181} = -22.6$ dB (black, dashed line in Fig. 6). The criterion (8) without Q -filter, shown in Fig. 6 by the black solid line, exceeds the allowed \mathcal{H}_∞ norm of -22.6 dB for frequencies $f > 228$ Hz.

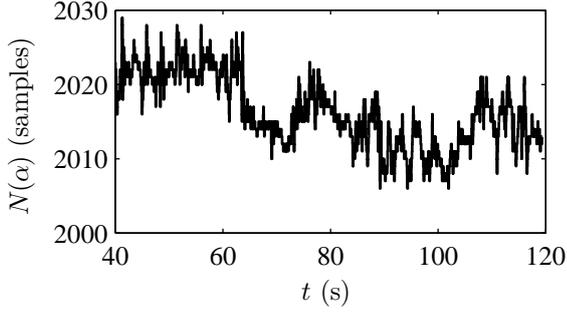


Fig. 5. Tracking error, PSD of the tracking error and variation in $N(\alpha)$ for an experiment without RC and with $\dot{r} = 10 \mu\text{m/s}$.

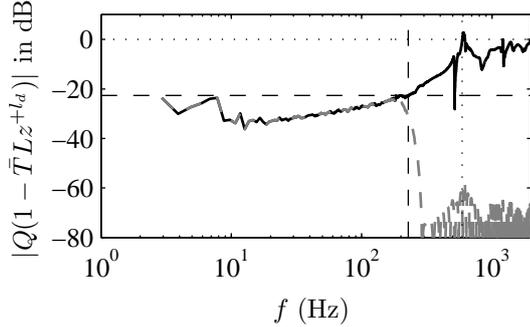


Fig. 6. Convergence criterion (4) without (black, solid) and with (grey, dashed) Q -filter.

To guarantee stability of DVRC, a low-pass $Q(z)$ FIR filter with 100 taps and a cut-off frequency of 220 Hz is used. With the robustness filter $Q(z)$ stability is guaranteed, as shown in Fig. 6 with the grey dashed line.

For comparison, a high-order repetitive controller that incorporates two periods, i.e., with two memory loops [13], is designed. The high-order repetitive controller equals $M_{HO}(z) = \frac{L(z)W(z)Q(z)z^{-(N-q_d-l_d)}}{1-Q(z)W(z)z^{-(N-q_d)}}$, where $W(z)$ is the high-order repetitive function $W(z) = \sum_{i=1}^{n_{HO}} w_i z^{-(i-1)N}$ and $n_{HO} = 2$ is the order. The optimal weighting filter for a second order repetitive controller is determined in [13] as $W_{opt} = (w_{opt,1}, w_{opt,2}) = (2, -1)$.

6 Experimental results

The steady-state tracking errors of the experiments with standard RC, DVRC and the high-order RC for $\dot{r} = 10 \mu\text{m/s}$ are shown in Fig. 7. The rms value of the tracking error without RC (top left figure) equals $\text{rms}(e(t)) = 109 \text{ nm}$. Standard RC reduces the tracking error to $\text{rms}(e_{RC}) = 18.3 \text{ nm}$ (top right figure in Fig. 7). Although the error is reduced significantly, a clear fluctuation in the magnitude of the error is visible, which is caused by the fact that the repetitive variable is not time. The remaining error with RC still contains a significant repetitive part. The high-order RC, shown in the bottom left figure of Fig. 7, reduces the tracking

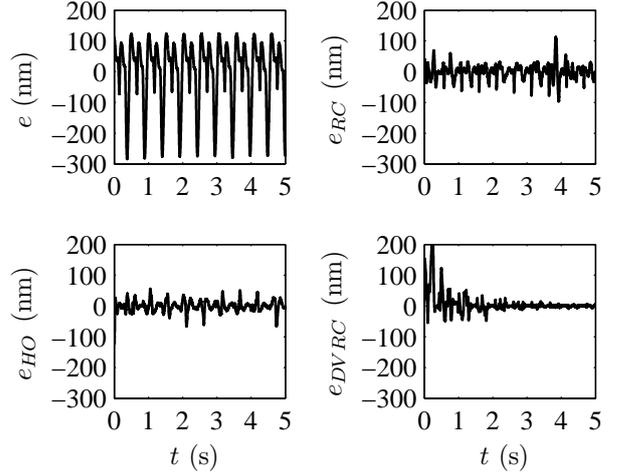


Fig. 7. Tracking errors of the experiments with $\dot{r} = 10 \mu\text{m/s}$ without RC, with RC, with high-order RC and with DVRC.

error further to $\text{rms}(e_{HO}) = 13.7 \text{ nm}$. The second order repetitive controller is not able to completely remove the fluctuation in the error, indicating that it is not able to cope with the amount of variation in the repetitive delay. Increasing the order of the repetitive controller would slightly increase the robustness to the variation, but requires a larger memory loop to incorporate an additional period. The tracking error with DVRC, shown in the bottom right figure of Fig. 7, significantly reduces the tracking error to $\text{rms}(e_{DVRC}) = 2.77 \text{ nm}$. DVRC reduces the tracking error by 97% compared to the tracking error without RC, by 85% compared to standard RC and by 80% compared to the high-order repetitive controller.

7 Conclusions

In this paper, we presented a delay-varying repetitive control (DVRC) method, which is applicable for systems that have a repetitive nature with respect to a repetitive variable other than time. DVRC uses knowledge of the repetitive variable of the system to determine and adjust the time-varying repetitive delay accordingly. We derived a new \mathcal{H}_∞ norm-based stability criterion for the proposed DVRC method. This criterion allows the design of the learning filters using frequency domain techniques as is common in RC. We showed that the developed DVRC method is able to significantly suppress the periodic disturbances in a nano-motion stage driven by a walking piezo actuator compared to existing methods.

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