Output-Based and Decentralized Dynamic Event-Triggered Control With Guaranteed $\mathcal{L}_p$-Gain Performance and Zeno-Freeness

V. S. Dolk, D. P. Borgers, and W. P. M. H. Heemels, *Fellow, IEEE*

***Abstract*—Networked control systems are often subject to limited communication resources. By only communicating output measurements when needed, event-triggered control is an adequate method to reduce the usage of communication resources while retaining desired closed-loop performance. In this work, a novel event-triggered control (ETC) strategy for a class of nonlinear feedback systems is proposed that can simultaneously guarantee a finite $\mathcal{L}_p$-gain and a strictly positive lower bound on the inter-event times. The new ETC scheme can be synthesized in an output-based and/or decentralized form, takes the specific medium access protocols into account, and is robust to (variable) transmission delays by design. Interestingly, in contrast with the majority of existing event-generators that only use static conditions, the newly proposed event-triggering conditions are based on dynamic elements, which has several advantages including larger average inter-event times. The developed theory leads to families of event-triggered controllers that correspond to different tradeoffs between (minimum and average) inter-event times, maximum allowable delays and $\mathcal{L}_p$-gains. A linear and a nonlinear numerical example will illustrate all the benefits of this new dynamic ETC scheme.**

***Index Terms*—Decentralized control, dynamic output-based control, event-triggered control systems, lyapunov methods, medium access protocols, networked control systems, $\mathcal{L}_p$ gains.**

I. Introduction

NETWORKED CONTROL SYSTEMS (NCSs) are distributed systems consisting of plants and controllers of which sensor and actuation data is transmitted over shared (wired or wireless) communication networks. These NCSs offer many advantages compared to conventional control systems in which sensor and actuation data is transmitted over dedicated point-to-point (wired) links. In particular, NCSs offer reduced installation costs, greater flexibility and better maintainability. Additionally, wireless communication is able to overcome the physical limitations of employing wired links. However, the usage of wireless communication also comes with inevitable network-induced imperfections. Indeed, since networked communication is inherently digital (packet-based), sensor and actuation data need to be quantized and cannot be transmitted continuously, but only at discrete time instants. Because the communication medium is often shared by multiple sensor, controller and actuator nodes, communication constraints are obviously also present. For this reason, there is a need for a medium access protocol, which governs the access of the nodes to the network, in order to prevent package dropouts, see, e.g., [1]–[4]. Furthermore, NCSs are typically subject to variable transmission delays which are in some cases further amplified by high occupation rates of the network.

In traditional (networked) control setups, the transmission instants are determined purely based on time. In fact, in standard digital sampled-data control the transmissions are often triggered periodically in time, possibly with some jitter and delays in the communication. This time-triggered communication approach is predictable and easy to implement. For these reasons, a large portion of the NCS literature aims at finding time-based specifications such that the closed-loop system meets the desired stability and performance criteria, see, e.g., [1]–[6]. However, a time-triggered approach often results in redundant transmissions, as many transmissions will occur at times when this is not actually needed to achieve the desired stability and performance properties. This is due to the fact that time-based specifications are typically determined via worst-case estimates and should hold for all situations (all states) the system can attain.

As an alternative, it seems more natural to let the transmission intervals depend on the state of the system, thereby determining the actual need of having to use the communication resources. This resource-aware control view seems more appealing to deal with the scarcity of communication resources, especially when the control systems share the communication network with other devices and users. Examples of resource-aware control strategies include self-triggered control, see [7]–[12], and event-triggered control, see [13]–[18] for some early approaches, and see [19] for a recent overview.

In an event-triggered control approach, the transmission times are determined on-line, using well-designed event conditions based on, e.g., output measurements of the system. As

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The authors are with the Control Systems Technology Group, Department of Mechanical Engineering, Eindhoven University of Technology, 5600 MB, Eindhoven, The Netherlands (e-mail: v.s.dolk@tue.nl; d.p.borgers@tue.nl; m.heemels@tue.nl).

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such, event-triggered control (ETC) is much better equipped than time-triggered control to balance resource utilization and control performance. One of the main difficulties of ETC is to design the *event-triggering mechanism* (ETM) in such a way that global asymptotic stability (GAS) (in absence of disturbances) and/or finite $\mathcal{L}_p$-stability (for $p \in [1, \infty)$) are guaranteed together with a *positive minimum inter-event time* (MIET), especially for output-based and decentralized control configurations and the situation where disturbances are present [20], [21]. This positive MIET is an essential property in order to exclude Zeno behavior (the occurrence of an infinite number of events in finite time), and to enable practical implementation of the ETC system. As such, an important problem is the construction of ETMs satisfying the following properties:

(i) Zeno-freeness in the sense of the existence of strictly positive MIETs, even in the presence of disturbances;
(ii) GAS in absence of disturbances and guaranteed $\mathcal{L}_p$-stability with finite $\mathcal{L}_p$-gains (for $p \in [1, \infty)$) with respect to a disturbance w and a certain performance output z.

Achieving these two requirements simultaneously is not trivial, as it has been shown recently in [20] that many ETMs do not lead to systems that have a positive MIET that is robust to disturbances, or do not guarantee GAS or $\mathcal{L}_p$-stability with finite $\mathcal{L}_p$-gains (where $p \in [1, \infty)$). For example, the approaches in [22] and [23] consider $\mathcal{L}_p$-gains, however they do not provide a strictly positive MIET for the case that $w \neq 0$.

Even in the absence of disturbances, inter-event times of many ETMs converge to zero in case of output-based and decentralized control configurations [20], [21]. For this reason, recent works on ETC employ either time regularization [15], [16], [24]–[32], in the sense that the triggering condition is only checked after a specific time duration δ since the last transmission has elapsed, or periodic event-triggered control (PETC) [15], [16], [33]–[36], in the sense that the triggering condition is checked at fixed periodic sampling time instants with sampling period h, such that the MIET is larger than or equal to δ or h, respectively. However, only a few of these works provide $\mathcal{L}_p$-stability analyses in case of output-based and decentralized event-triggered control. These include [33], [34], and [36], all using PETC schemes.

In this paper, building upon the work of [2] and [6] and extending our preliminary work [37] that focussed on the centralized state-feedback case only, a novel time-regularized *event-triggered control* strategy is introduced that does satisfy the above mentioned criteria. Our new method uses the unique combination of *dynamic* event-triggering conditions [38], [39] and time regularization [16], [24]–[28], [31]–[33]. Unlike [38] and [39], which consider the state-feedback case where neither delays nor disturbances are present, here we consider the output-based and/or decentralized case with delays and disturbances. The use of dynamic ETMs is a key ingredient, as *static* ETMs (which are studied in the majority of the ETC literature) in combination with time-regularization, often reduce to approximately time-triggered periodic communication, in presence disturbances when the state is close to the origin which typically leads to redundant transmission instants, see, e.g., [20, Ex. 3] or the numerical example in [37]. Under the same stability and performance properties, our dynamic ETC scheme does not exhibit this undesirable behavior and satisfies next to (i) and (ii) also the following properties:

(iii) when the system is close to the desired equilibrium, the transmission instants do not unnecessarily become (almost) equidistantly distributed in time with inter-transmission intervals close to the enforced lower bounds δ or h, as mentioned before;
(iv) robust performance in the sense that the guarantees on GAS and/or the $\mathcal{L}_p$-gains also hold in the presence of (variable) transmission delays (within certain bounds);
(v) output-based and/or decentralized (asynchronous and multi-network) form (in the sense that multiple networks can be present in the system).

A starting point of our results is the work [2], which leads in the context of NCSs to *time-based* specifications in terms of a so-called *maximum allowable transmission interval* (MATI) and a *maximum allowable delay* (MAD). The resulting tradeoff curves in [2] specify that as long as the transmission intervals are smaller than the MATI and the delays are smaller than the (corresponding) MAD, specific upper bounds on the $\mathcal{L}_p$-gains are guaranteed. Hence, the MATI and the MAD are bounds used to express the timing specifications. Interestingly, for a given $\mathcal{L}_p$-gain, the MIET and the MAD of the ETC strategy proposed here are close to or equal to the MATI and MAD in [2] and [6] corresponding to the same $\mathcal{L}_p$-gain. However, simulations show that the average transmission interval of the proposed strategy is much larger due to the event-based triggering. Hence, the ETC method also achieves the following property, which is a general requirement for any ETC scheme:

(vi) effectively achieve the same control performance, but significantly reduce the number of transmissions compared to control strategies using time-based specifications, with the MIET close to the available MATI bounds but the average inter-event times much larger.

Although we do not prove this analytically in the present paper, we show through numerical examples that the proposed dynamic ETC scheme indeed displays this property in situations where the conventional (static) ETC schemes fail. In fact, the newly proposed ETC scheme satisfies all the six criteria (i)–(vi), which is, to the best of our knowledge, the first time that this has been accomplished. Moreover, the corresponding event-triggering conditions can be constructed via a systematic procedure allowing tradeoffs among performance in terms of $\mathcal{L}_p$-gains, robustness in terms of MADs, and network utilization, which will be demonstrated by means of two numerical examples.

The remainder of this paper is organized as follows. After presenting the necessary preliminaries and notational conventions in Section II, we introduce the decentralized networked control setup and the problem statement in Section III, which are formalized in Section IV by deriving a complete mathematical model of the closed-loop system. In Section V we derive conditions for the proposed event-triggering strategy such that stability or $\mathcal{L}_p$-gain properties can be guaranteed. Section VI shows how an ETM can be designed systematically and implemented. Finally, we illustrate the presented theory with two numerical examples in Section VII, and provide conclusions in Section VIII.

II. Definitions and Preliminaries

The following notational conventions are used in this paper. $\mathbb{N}$ denotes the set of all non-negative integers, $\mathbb{N}_{>0}$ denotes the set of all positive integers, $\mathbb{R}$ denotes the field of all real numbers and $\mathbb{R}_{\geqslant 0}$ denotes the set of all non-negative reals. For $N \in \mathbb{N}$, we write the set $\{1, 2, \ldots, N\}$ as $\bar{N}$. For N vectors $x_i \in \mathbb{R}^{n_i}, i \in \bar{N}$, we denote the vector obtained by stacking all vectors in one (column) vector $x \in \mathbb{R}^n$ with $n = \sum_{i=1}^{N} n_i$ by $(x_1, x_2, \ldots, x_N)$, i.e., $(x_1, x_2, \ldots, x_N) = [x_1^\top, x_2^\top, \ldots, x_N^\top]^\top$. The vectors in $\mathbb{R}^N$ consisting of all ones and zeros are denoted by $\mathbf{1}_N$ and $\mathbf{0}_N$, respectively. By $|\cdot|$ and $\langle \cdot, \cdot \rangle$ we denote the Euclidean norm and the usual inner product of real vectors, respectively. With $\bar{e}_i$ we denote the compound vector of all $e_j, j \in \bar{N} \setminus \{i\}$, i.e., $\bar{e}_i = (e_1, \ldots, e_{i-1}, e_{i+1}, \ldots, e_N)$. For a real symmetric matrix A, $\lambda_{\max}(A)$ denotes the largest eigenvalue of A. I_N denotes the identity matrix of dimension $N \times N$ and, if N is clear from the context, we write I. $0_{N \times M}$ denotes an $N \times M$ matrix with all entries equal to zero. The expression $\mathrm{diag}(A_1, A_2, \ldots, A_N)$ with matrices $A_i \in \mathbb{R}^{n_i \times m_i}$ denotes a block-diagonal matrix. A function $\alpha : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is said to be of class $\mathcal{K}$ if it is continuous, strictly increasing and $\alpha(0) = 0$. It is said to be of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and it is unbounded. A continuous function $\beta : \mathbb{R}_{\geqslant 0} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is said to be of class $\mathcal{KL}$ if, for each fixed s, the mapping $r \mapsto \beta(r, s)$ belongs to class $\mathcal{K}$ and for each fixed r, the mapping $s \to \beta(r, s)$ is decreasing and $\beta(r, s) \to 0$ as $s \to \infty$. A continuous function $\gamma : \mathbb{R}_{\geqslant 0} \times \mathbb{R}_{\geqslant 0} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is said to be of class $\mathcal{KLL}$ if, for each $r \geqslant 0$, both $\gamma(\cdot, \cdot, r)$ and $\gamma(\cdot, r, \cdot)$ belong to class $\mathcal{KL}$. A function $f : \mathbb{R}^n \to \mathbb{R}^n$ is said to be locally Lipschitz continuous if for each $x_0 \in \mathbb{R}^n$ there exist constants $\delta > 0$ and $L > 0$ such that for all $x \in \mathbb{R}^n$ we have that $|x - x_0| \leqslant \delta \Rightarrow |f(x) - f(x_0)| \leqslant L|x - x_0|$. A function $f : \mathbb{R}^n \to \mathbb{R}^n$ is said to be linearly bounded if there exists a constant $L > 0$ such that for all $x \in \mathbb{R}^n$, we have that $|f(x)| \leqslant L|x|$.

In this paper, we will model network control systems (NCSs) as a hybrid system $\mathcal{H}$ of the form

$$\dot{\xi} = F(\xi, w), \quad \text{when } \xi \in C \tag{1a}$$

$$\xi^+ \in G(\xi), \quad \text{when } \xi \in D \tag{1b}$$

where F describes the flow dynamics, G the jump dynamics, C the flow set, and D the jump set. We now recall some definitions given in [40] on the solutions of such hybrid system.

Definition 1: A compact hybrid time domain is a set $\mathcal{D} = \bigcup_{j=0}^{J-1} [t_j, t_{j+1}] \times \{j\} \subset \mathbb{R}_{\geqslant 0} \times \mathbb{N}$ with $J \in \mathbb{N}_{>0}$ and $0 = t_0 \leqslant t_1, \ldots, \leqslant t_J$. A hybrid time domain is a set $\mathcal{D} \subset \mathbb{R}_{\geqslant 0} \times \mathbb{N}$ such that $\mathcal{D} \cap ([0, T] \times \{0, 1, \ldots, J\})$ is a compact hybrid time domain for each $(T, J) \in \mathcal{D}$.

Definition 2: A hybrid trajectory is a pair $(\mathrm{dom}\,\xi, \xi)$ consisting of a hybrid time domain $\mathrm{dom}\,\xi$ and a function ξ defined on $\mathrm{dom}\,\xi$ that is absolutely continuous in t on $(\mathrm{dom}\,\xi) \cap (\mathbb{R}_{\geqslant 0} \times \{j\})$ for each $j \in \mathbb{N}$.

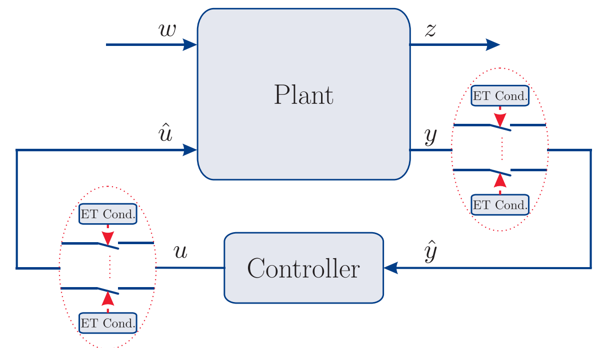

Fig. 1. Networked control setup consisting of a plant $\mathcal{P}$ and a controller $\mathcal{C}$, connected via N independent communication networks $\mathcal{N}_i$, $i \in \bar{N}$.

Definition 3: For the hybrid system $\mathcal{H}$ given by the state space $\mathbb{R}^n$, the (disturbance) input space $\mathbb{R}^{n_w}$ and the data (F, G, C, D), where the flow map $F : \mathbb{R}^n \times \mathbb{R}^{n_w} \to \mathbb{R}^n$, the jump map $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$, and the flow set C and the jump set D are subsets of $\mathbb{R}^n$, a hybrid trajectory $(\mathrm{dom}\,\xi, \xi)$ with $\xi : \mathrm{dom}\,\xi \to \mathbb{R}^n$ is a solution to $\mathcal{H}$ for a locally integrable input function $w : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^{n_w}$, if the following is provided:

1) for all $j \in \mathbb{N}$ and for almost all $t \in I_j := \{t \in \mathbb{R}_{\geqslant 0} | (t, j) \in \mathrm{dom}\,\xi\}$, we have $\xi(t, j) \in C$ and $\dot{\xi}(t, j) = F(\xi(t, j), w(t))$;
2) for all $(t, j) \in \mathrm{dom}\,\xi$ such that $(t, j+1) \in \mathrm{dom}\,\xi$, we have $\xi(t, j) \in D$ and $\xi(t, j+1) \in G(\xi(t, j))$.

For the motivation and more details on these definitions, the interested reader is referred to [40]. We will often not mention $\mathrm{dom}\,\xi$ explicitly, and understand that with each hybrid trajectory ξ comes a hybrid time domain $\mathrm{dom}\,\xi$.

In addition, for $p \in \mathbb{R}_{\geqslant 1}$, we introduce the $\mathcal{L}_p$-norm of a function ξ defined on a hybrid time domain $\mathrm{dom}\,\xi = \bigcup_{j=0}^{J-1} [t_j, t_{j+1}] \times \{j\}$ with J possibly ∞ and/or $t_J = \infty$ by

$$\|\xi\|_{\mathcal{L}_p} = \left(\sum_{j=0}^{J-1} \int_{t_j}^{t_{j+1}} |\xi(t, j)|^p \, dt \right)^{\frac{1}{p}} \tag{2}$$

provided the right-hand side is well-defined and finite. In case $\|\xi\|_{\mathcal{L}_p}$ is finite, we say that $\xi \in \mathcal{L}_p$. Note that this definition is essentially identical to the usual $\mathcal{L}_p$-norm in case a function is defined on a subset of $\mathbb{R}_{\geqslant 0}$.

Lemma 1: Consider $a, b \in \mathbb{R}$ and some constant $\varepsilon > 0$, then it holds that $2ab \leqslant (1/\varepsilon)a^2 + \varepsilon b^2$.

III. NCS Setup and Problem Statement

In this section, we introduce the (decentralized) event-triggered NCS with communication constraints caused by network limitations and network-induced imperfections such as varying transmission delays. Based on this description, we also provide the problem statement considered in this paper.

A. Networked Control Configuration

In this paper, we consider the networked control configuration shown in Fig. 1 which consists of a continuous-time nonlinear plant $\mathcal{P}$ and a controller $\mathcal{C}$ which are connected via N

independent communication networks. In an NCS, in contrast to conventional control setups that rely on dedicated point-to-point links, the sensor and/or actuator data is communicated over (wireless) networks in a package-based manner. The plant $\mathcal{P}$ is given by

$$\mathcal{P} : \begin{cases} \dot{x}_p &= f_p(x_p, \hat{u}, w) \\ y &= g_p(x_p) \end{cases} \tag{3}$$

where $x_p \in \mathbb{R}^{n_p}$ denotes the plant state, $w \in \mathbb{R}^{n_w}$ is a disturbance input, $\hat{u} \in \mathbb{R}^{n_u}$ represents a vector of the most recently received control signals and $y \in \mathbb{R}^{n_y}$ is the output of $\mathcal{P}$. The controller $\mathcal{C}$ is given by

$$\mathcal{C} : \begin{cases} \dot{x}_c &= f_c(x_c, \hat{y}) \\ u &= g_c(x_c) \end{cases} \tag{4}$$

where $x_c \in \mathbb{R}^{n_c}$ denotes the controller state, $\hat{y} \in \mathbb{R}^{n_y}$ represents a vector of most recently received sensor data, and $u \in \mathbb{R}^{n_u}$ represents a vector of control signals. The functions f_p and f_c are assumed to be continuous and the functions g_p and g_c are assumed to be continuously differentiable.

The communication of sensor and actuator data between $\mathcal{P}$ and $\mathcal{C}$ is (possibly) performed via multiple networks $\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_N$ that operate asynchronously and independently. To facilitate the description and analysis of the decentralized event-triggered control setup, we define $v := (y, u) \in \mathbb{R}^{n_v}$ and $\hat{v} := (\hat{y}, \hat{u}) \in \mathbb{R}^{n_v}$ with $n_v := n_y + n_u$. For simplicity of exposition, we assume (possibly after reordering) that $v = (v_1, v_2, \ldots, v_N)$ and $\hat{v} = (\hat{v}_1, \hat{v}_2, \ldots, \hat{v}_N)$ and that $v_i, i \in \bar{N}$, is communicated over the network $\mathcal{N}_i, i \in \bar{N}$.

B. Communication Networks and Protocols

In the networked control configuration of Fig. 1, output v_i is sampled and transmitted over the network $\mathcal{N}_i, i \in \bar{N}$, to the plant $\mathcal{P}$ or controller $\mathcal{C}$, at transmission times $t_j^i, j \in \mathbb{N}$, satisfying $0 \leqslant t_0^i < t_1^i < t_2^i <, \ldots$. The update of $\hat{v}_i$ corresponding to transmission time t_j^i occurs after a communication delay of τ_j^i time units. See [2] for a more detailed discussion on communication delays.

In this paper, we consider the so-called small-delay case, meaning that reception of new information occurs before the next transmission is due. In fact, we assume that the communication delays in each network $\mathcal{N}_i, i \in \bar{N}$, are upper-bounded by a *maximum allowable delay* (MAD). To be more precise, we adopt the following standing assumption.

Standing Assumption 1: The transmission delays in network $\mathcal{N}_i, i \in \bar{N}$, satisfy $0 \leqslant \tau_j^i \leqslant \tau_{\text{mad}}^i \leqslant \tau_{\text{miet}}^i$, for all $j \in \mathbb{N}$, where τ_{mad}^i denotes the MAD of network $\mathcal{N}_i$ and where τ_{miet}^i denotes a lower-bound on the *minimum inter-event time* of network $\mathcal{N}_i$, i.e., $\tau_{\text{miet}}^i \leqslant \inf_{j \in \mathbb{N}} (t_{j+1}^i - t_j^i)$.

An update of $\hat{v}_i$ at $t_j^i + \tau_j^i$ for some $i \in \bar{N}$ and $j \in \mathbb{N}$, can be described as

$$\hat{v}_i \left(\left(t_j^i + \tau_j^i \right)^+ \right) = v_i \left(t_j^i \right) + h_i \left(j, e_i \left(t_j^i \right) \right) \tag{5}$$

where $e_i \in \mathbb{R}^{n_{v,i}}$ denotes the network-induced error $e_i := \hat{v}_i - v_i$ and where $e := \hat{v} - v = (e_1, e_2, \ldots, e_N)$. The functions $h_i, i \in \bar{N}$, can be used to model medium access protocols as described in [1], [2], and [6]. These medium access protocols are typically employed in case network $\mathcal{N}_i$ is shared by multiple (sensor or actuator) nodes where each of these nodes correspond to a subset of the entries of $v_i/\hat{v}_i, i \in \bar{N}$.

Remark 1: In the ETC literature, the standard case that is typically considered is that each network updates asynchronously according to $\hat{v}_i((t_j^i + \tau_j^i)^+) = v_i(t_j^i)$, which corresponds to $h_i(j, e_i) = 0$, for all $i \in \bar{N}$, $j \in \mathbb{N}$, and all $e_i \in \mathbb{R}^{n_{v,i}}$. However, by building upon [1], [2], and [6], a framework can be obtained that allows us to study next to this standard sampled-data ETC setup, also other access protocols including the round-robin (RR) and the try-once-discard (TOD) protocols in one framework without additional burden. Therefore, we decided to present the work at this level of generality. We envision that the combination of ETC and medium access protocols could be valuable in future extensions and particular applications.

Remark 2: For simplicity of exposition, we focus here on the control configuration as in Fig. 1. However, the ETC method presented in this work also applies to other configurations such as the case of static state feedback control and the case where either $\hat{u} = u$ or $\hat{y} = y$, meaning that the corresponding signals are not transmitted over a (shared) network, but are continuously available. These configurations can all be captured in the same hybrid model as presented in Section IV-B on which our ETC design is based. To illustrate this flexibility, we will consider two numerical examples in which $\hat{u} = u$ in Section VII.

C. Problem Statement

As motivated in the introduction, in most NCSs it is desirable to reduce the usage of communication resources as much as possible (while still guaranteeing the required stability and performance properties). Therefore, we consider event-triggered communication as illustrated in Fig. 1.

The problem considered in this work is to derive a systematic methodology for the design of triggering conditions such that the criteria (i)–(vi) discussed in the introduction are met. A more rigorous problem formulation is provided in the next section, based on a complete mathematical model for the event-triggered closed-loop NCS.

IV. Mathematical Model of the Event-Triggered Control Setup

A. Event-Triggering Mechanism

In this work, for each network $\mathcal{N}_i, i \in \bar{N}$, the proposed event-triggering mechanism takes the form

$$t_0^i = 0, \ t_{j+1}^i := \inf \left\{ t > t_j^i + \tau_{\text{miet}}^i | \eta_i(t) \leqslant 0 \right\} \tag{6}$$

for $i \in \bar{N}$ and for all $j \in \mathbb{N}$, where $\tau_{\text{miet}}^i \in \mathbb{R}_{>0}$ is a lower bound on the MIET (see also Standing Assumption 1) and $\eta_i \in \mathbb{R}_{\geqslant 0}$ is an auxiliary variable. The variable η_i will evolve according to

$$\dot{\eta}_i = \Psi_i(o_i) \tag{7}$$

for $i \in \bar{N}$, for some well-designed function Ψ_i, which we will specify in Section V. Let us already remark that the argument o_i

represents locally available information only, such as v_i, e_i and some other local variables, which are introduced in Section IV-B. In this manner, the event generators will have a decentralized and output-based structure and thereby enable satisfaction of property (v) mentioned in the introduction. Note that the state variables $\eta_i, i \in \bar{N}$, evolve independently of each other and thus the transmissions for the networks $\mathcal{N}_i, i \in \bar{N}$, are indeed triggered asynchronously and independently. Observe that by taking $\tau^i_{\text{miet}} \in \mathbb{R}_{>0}, i \in \bar{N}$, the adopted time regularization imposes that the next event can only take place after at least a fixed amount of time $\tau^i_{\text{miet}} > 0$ has elapsed, i.e., $t^i_{j+1} - t^i_j \geqslant \tau^i_{\text{miet}}$, for each $j \in \mathbb{N}$. In this way, so-called Zeno behavior is excluded from the ETC system and property (i) mentioned in the introduction is satisfied. Moreover, observe that Standing Assumption 1 is indeed valid for the event-triggered system if τ^i_{miet} is designed such that $\tau^i_{\text{miet}} \geqslant \tau^i_{\text{mad}}$, for all $i \in \bar{N}$. As we will show in Section V, the time τ^i_{miet} can be taken close to or, under some conditions, equal to the MATI bounds as derived in [1], [2], [4], [6]. As such, the smallest inter-transmission times (lower bounded by the MIET) in the ETC scheme are always close to or equal to the largest inter-transmission times (MATI) in time-based triggering with the consequence that the resource utilization of the ETC scheme will never be worse than the triggering based on time-based specifications (MATI), as in [1], [2], [4], [6]. In addition, we will show that the average inter-transmission times are typically much larger in the novel ETC schemes that we will propose. This corresponds to property (vi) mentioned in the introduction. Finally, notice that a *dynamic event-triggering mechanism* (6), (7) is employed which, as we will discuss in Remark 4 and show in the numerical examples, is important in realizing property (iii) mentioned in the introduction. The satisfaction of properties (ii) and (iv) follow from the analysis in Section V.

B. Hybrid Model

In order to analyze asymptotic stability and $\mathcal{L}_p$-stability in the next section, we model the NCS employing event-triggered communication by means of the hybrid system framework as developed in [40] and [41], which was also employed in [1], [2], [6], [27], [28], [39], [42] in the context of NCSs.

To provide this hybrid model, we assume that the value of $\hat{v}_i$ evolves according to a zero-order hold model, meaning that $\hat{v}_i$ is kept constant between the update times $t^i_j + \tau^i_j$ and $t^i_{j+1} + \tau^i_{j+1}$ for all $j \in \mathbb{N}$, i.e.,

$$\dot{\hat{v}}_i(t) = 0, \quad t \in \left(t^i_j + \tau^i_j, t^i_{j+1} + \tau^i_{j+1}\right]. \tag{8}$$

By means of (5), following [2], we can write the jump dynamics of e_i at an update event as follows:

$$\begin{aligned} e_i\left(\left(t^i_j + \tau^i_j\right)^+\right) &= \hat{v}_i\left(\left(t^i_j + \tau^i_j\right)^+\right) - v_i\left(t^i_j + \tau^i_j\right) \\ &= h_i\left(j, e_i\left(t^i_j\right)\right) + v_i\left(t^i_j\right) - \hat{v}_i\left(t^i_j\right) \\ &\quad - v_i\left(t^i_j + \tau^i_j\right) + \hat{v}_i\left(t^i_j + \tau^i_j\right) \\ &= h_i\left(j, e_i\left(t^i_j\right)\right) - e_i\left(t^i_j\right) + e_i\left(t^i_j + \tau^i_j\right). \end{aligned} \tag{9}$$

Notice that we used the fact that $\hat{v}_i(t^i_j) = \hat{v}_i(t^i_j + \tau^i_j)$, for each $j \in \mathbb{N}$, due to the zero-order hold assumption and Standing Assumption 1. Here we consider $\hat{v}_i$ and e_i as left-continuous signals in the sense that for all $t > 0$, $\hat{v}_i(t) = \lim_{s \uparrow t} \hat{v}_i(s)$.

In order to formulate the dynamics of the event-triggered NCS in terms of flow and jump equations as in [40] and [41], we will introduce as in [2] additional states $l_i \in \{0, 1\}$, $s_i \in \mathbb{R}^{n_i}$, $\kappa_i, \in \mathbb{N}$, and $\tau_i \in \mathbb{R}_{\geqslant 0}$, $i \in \bar{N}$. The variable l_i is a boolean, which keeps track of whether the next event in network $\mathcal{N}_i$ is a transmission event or an update event, indicated by $l_i = 0$ and $l_i = 1$, respectively. The variable s_i serves as a memory variable to store the value $h_i(j, e_i(t^i_j)) - e_i(t^i_j)$ (as present in (9)) at the moment of a transmission at time t^i_j and is used to model the update event at time $t^i_j + \tau^i_j$. The integer variable κ_i is used to keep track of the total amount of transmissions in network $\mathcal{N}_i$ over time and the timer variable τ_i, $i \in \bar{N}$, is adopted to capture the time elapsed since the last transmissions and to assure that Standing Assumption 1 remains valid. Consider the state vector $\xi := (x, e, \tau, \kappa, s, l, \eta) \in \mathbb{X}$ with $\mathbb{X} := \{(x, e, \tau, \kappa, s, l, \eta) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_v} \times \mathbb{R}^N_{\geqslant 0} \times \mathbb{N}^N \times \mathbb{R}^{n_v} \times \{0, 1\}^N \times \mathbb{R}^N_{\geqslant 0}\}$, and the vectors $\tau := (\tau_1, \tau_2, \ldots, \tau_N) \in \mathbb{R}^N_{\geqslant 0}$, $\kappa := (\kappa_1, \kappa_2, \ldots, \kappa_N) \in \mathbb{N}^N$, $s := (s_1, s_2, \ldots, s_N) \in \mathbb{R}^{n_v}$, $l := (l_1, l_2, \ldots, l_N) \in \{0, 1\}^N$, and $\eta := (\eta_1, \eta_2, \ldots, \eta_N) \in \mathbb{R}^N_{\geqslant 0}$. By combining these new state variables with (3), (4), and (7), the flow dynamics of the interconnection $(\mathcal{P}, \mathcal{C}, \mathcal{N}_1, \ldots, \mathcal{N}_N)$ is given by

$$\begin{aligned} F(\xi, w) := (&f(x, e, w), g(x, e, w), \mathbf{1}_N, \mathbf{0}_N, \mathbf{0}_N, \mathbf{0}_N, \\ &\Psi(v, e, \tau, \kappa, s, l, \eta)) \end{aligned} \tag{10}$$

where $\Psi(v, e, \tau, \kappa, s, l, \eta) = (\Psi_1(o_1), \Psi_2(o_2), \ldots, \Psi_N(o_N))$ with $o_i = (v_i, e_i, \tau_i, \kappa_i, s_i, l_i, \eta_i) \in \mathbb{O}_i := \mathbb{R}^{n_{v,i}} \times \mathbb{R}^{n_{v,i}} \times \mathbb{R}_{\geqslant 0} \times \mathbb{N} \times \mathbb{R}^{n_{v,i}} \times \{0, 1\} \times \mathbb{R}_{\geqslant 0}$, and where $f(x, e, w)$ is given by

$$f(x, e, w) = \begin{bmatrix} f_p\left(x_p, g_c(x_c) + \Gamma_u e, w\right) \\ f_c\left(x_c, g_p(x_p) + \Gamma_y e\right) \end{bmatrix} \tag{11}$$

where $\Gamma_y := [I_{n_y} 0_{n_y \times n_u}]$ and $\Gamma_u := [0_{n_u \times n_y} I_{n_u}]$. To obtain the expression for $g(x, e, w)$, observe that due to (8) we have that $\dot{e} = -\dot{v}$. By combining this fact with (3) and (4), we obtain that

$$g(x, e, w) = \begin{bmatrix} -\frac{\partial g_p}{\partial x_p}(x_p) f_p\left(x_p, g_c(x_c) + \Gamma_u e, w\right) \\ -\frac{\partial g_c}{\partial x_c}(x_c) f_c\left(x_c, g_p(x_p) + \Gamma_y e\right) \end{bmatrix}. \tag{12}$$

The corresponding flow set is given by

$$C := \bigcap_{i \in \bar{N}} C_i \tag{13}$$

with

$$\begin{aligned} C_i := \{\xi \in \mathbb{X} |\, &\left(\left(\tau_i \leqslant \tau^i_{\text{miet}} \vee \eta_i \geqslant 0\right) \wedge l_i = 0\right) \\ &\vee \left(0 \leqslant \tau_i \leqslant \tau^i_{\text{mad}} \wedge l_i = 1\right)\} \end{aligned} \tag{14}$$

for $i \in \bar{N}$.

To define the jump map, we introduce the notation $\Gamma_i \in \mathbb{R}^{N \times N}$, representing a matrix of which the ii-th (diagonal) entry is equal to one and all other entries are zero, and $\bar{\Gamma}_i \in \mathbb{R}^{n_v \times n_v}$ is a diagonal matrix consisting of diagonal elements being zero for the indices corresponding to the networks $\mathcal{N}_1, \ldots, \mathcal{N}_{i-1}$, $\mathcal{N}_{n+1}, \ldots, \mathcal{N}_N$ and one for the network $\mathcal{N}_i$. In particular

$$(\bar{\Gamma}_i)_{jj} = \begin{cases} 1, & \sum_{l=1}^{i-1} n_{v,l} < j \leqslant \sum_{l=1}^{i} n_{v,l} \\ 0, & \text{otherwise.} \end{cases}$$

The jump dynamics is given by $\xi^+ \in G(\xi)$ with jump map $G(\xi) = \bigcup_{i=1}^N G_i(\xi)$, where

$$G_i(\xi) := \begin{cases} \{G_{0,i}(\xi)\}, & \text{when } \xi \in D_i \wedge l_i = 0 \\ \{G_{1,i}(\xi)\}, & \text{when } \xi \in D_i \wedge l_i = 1 \\ \emptyset, & \text{when } \xi \notin D_i \end{cases} \tag{15}$$

$$G_{0,i}(\xi) := \begin{pmatrix} x \\ e \\ (I_N - \Gamma_i)\tau \\ \kappa + \Gamma_i \mathbf{1}_N \\ \bar{\Gamma}_i(h(\kappa, e) - e) + \left(I_{n_v} - \bar{\Gamma}_i\right) s \\ l + \Gamma_i \mathbf{1}_N \\ \eta \end{pmatrix} \tag{16}$$

$$G_{1,i}(\xi) := \left(x, \bar{\Gamma}_i s + e, \tau, \kappa, \left(I_{n_v} - \bar{\Gamma}_i\right) s, l - \Gamma_i \mathbf{1}_N, \eta\right) \tag{17}$$

where $h(\kappa, e) = (h_1(\kappa_1, e_1), h_2(\kappa_2, e_2), \ldots, h_N(\kappa_N, e_N))$. The function $G_{0,i}$ describes how the entire state ξ jumps when network $\mathcal{N}_i$ grants access to one of its nodes to transmit. Note that the memory state s_i is assigned the value $h_i(\kappa_i, e_i) - e_i$, which corresponds to the term $h_i(j, e_i(t_j^i)) - e_i(t_j^i)$ in (9), while the transmission error e_i remains the same. Furthermore, observe that l_i jumps to the value 1, which ensures that the next event in network $\mathcal{N}_i$ can only be an update event (thereby guaranteeing the small-delay assumption as described in Standing Assumption 1). The function $G_{1,i}$ describes how ξ jumps when in $\mathcal{N}_i$ an update event occurs. Observe that the reset of e_i as described by this part of the jump map corresponds to (9) and that l_i toggles back to 0, enforcing a transmission event as the next event in network $\mathcal{N}_i$. Also important to note is that the jump for s_i when $l_i = 1$ can be chosen arbitrarily from a modeling point of view since no information needs to be stored at the moment of an update event. We arbitrate here the update $s_i^+ = 0$. The corresponding jump set is given by

$$D := \bigcup_{i \in \bar{N}} D_i \tag{18}$$

where

$$D_i := \left\{\xi \in \mathbb{X} | \left(\tau_i \geqslant \tau_{\text{miet}}^i \wedge \eta_i \leqslant 0 \wedge l_i = 0\right) \vee (l_i = 1)\right\}. \tag{19}$$

By means of definitions (10)–(19) presented above, we can now obtain the hybrid model $\mathcal{H} = (F, G, C, D)$ as in (1) representing the entire decentralized ETC setup as illustrated in Fig. 1.

Definition 4: For the hybrid system $\mathcal{H}$ with $w = 0$, the set given by $\mathcal{E} := \{\xi \in \mathbb{X} | x = \mathbf{0}_{n_x}, e = s = \mathbf{0}_{n_v}, \eta = \mathbf{0}_N\}$ is said to be uniformly globally asymptotically stable (UGAS) if there exists a function $\beta \in \mathcal{KLL}$ such that, for any initial condition $\xi(0,0) \in \mathbb{X}_0$ with $\mathbb{X}_0 = \{\xi \in \mathbb{X} | l = \mathbf{0}_N\}$, all corresponding solutions ξ as in Definition 3 satisfy

$$|(x(t,j), e(t,j), s(t,j), \eta(t,j))| \leqslant \beta\left(|(x(0,0), e(0,0), s(0,0), \eta(0,0))|, t, j\right)$$

for all $(t, j) \in \operatorname{dom} \xi$. The set $\mathcal{E}$ is said to be uniformly globally exponentially stable (UGES) if β can be taken of the form $\beta(r, t, k) = Mr\exp(-\varrho(t + k))$ for some $M \geqslant 0$ and $\varrho > 0$ in the inequality above.

In case of the disturbance w being present, the performance of the hybrid system $\mathcal{H}$ might be defined as the level of disturbance attenuation with respect to some output variable

$$z = q(x, w). \tag{20}$$

Definition 5: The hybrid system $\mathcal{H}$ is said to be $\mathcal{L}_p$-stable from input w to output z with an $\mathcal{L}_p$-gain less than or equal to θ, if there exists a $\mathcal{K}_\infty$-function β such that for any exogenous input $w \in \mathcal{L}_p$, and any initial condition $\xi(0,0) \in \mathbb{X}_0$ with $\mathbb{X}_0 = \{\xi \in \mathbb{X} | l = \mathbf{0}_N\}$, each corresponding solution to $\mathcal{H}$ satisfies

$$\|z\|_{\mathcal{L}_p} \leqslant \beta\left(|(x(0,0), e(0,0), s(0,0), \eta(0,0))|\right) + \theta \|w\|_{\mathcal{L}_p}. \tag{21}$$

The problem that we loosely formulated at the end of Section III can now be formally given as follows.

Problem 1: Given a controller (4) for the plant (3) and a desired $\mathcal{L}_p$-gain $\theta \in \mathbb{R}_{\geqslant 0}$ in case disturbances are present, determine the values of $\tau_{\text{miet}}^i \in \mathbb{R}_{>0}$, the function Ψ^i in the event generator given by (6) and (7), and values of the maximum allowable delays τ_{mad}^i, for each network $\mathcal{N}_i, i \in \bar{N}$, such that the system $\mathcal{H}$ is UGAS (and sometimes even UGES) in case $w = 0$ and/or, in the presence of disturbances, $\mathcal{L}_p$-stable with an $\mathcal{L}_p$-gain less than or equal to θ, with a strictly positive τ_{miet}^i to assure Zeno-freeness and with large (average) inter-event times $t_{j+1}^i - t_j^i$, $j \in \mathbb{N}$.

V. ETM Design Conditions With Stability and $\mathcal{L}_p$-Gain Guarantees

In Section V-A and B, conditions are presented such that the ETM given by (6) and (7) ensures UGAS (and sometimes even UGES) in case $w = 0$ and $\mathcal{L}_p$-stability with a desired $\mathcal{L}_p$-gain in case disturbances are present for the system $\mathcal{H}$. In both cases, in order to specify suitable $\tau_{\text{miet}}^i, \tau_{\text{mad}}^i$ and Ψ_i, for all $i \in \bar{N}$, for the ETM given by (6) and (7), we need to consider the following condition.

Condition 1 [2]: For each $i \in \bar{N}$, there exist a function $\tilde{W}_i : \mathbb{N} \times \{0,1\} \times \mathbb{R}^{n_{v,i}} \times \mathbb{R}^{n_{v,i}} \to \mathbb{R}_{\geqslant 0}$ with $\tilde{W}_i(\kappa_i, l_i, \cdot, \cdot)$ locally Lipschitz for all fixed $\kappa_i \in \mathbb{N}$ and $l_i \in \{0,1\}$, $\mathcal{K}_\infty$-functions $\underline{\beta}_{W,i}$ and $\bar{\beta}_{W,i}$, and $0 < \lambda_i < 1$ such that for all $\kappa_i \in \mathbb{N}$, $l_i \in \{0,1\}$, $s_i \in \mathbb{R}^{n_{v,i}}$ and all $e_i \in \mathbb{R}^{n_{v,i}}$, it holds that

$$\tilde{W}_i\left(\kappa_i + 1, 1, e_i, h_i(\kappa_i, e_i) - e_i\right) \leqslant \lambda_i \tilde{W}_i(\kappa_i, 0, e_i, s_i) \tag{22}$$

$$\tilde{W}_i(\kappa_i, 0, s_i + e_i, 0) \leqslant \tilde{W}_i(\kappa_i, 1, e_i, s_i) \tag{23}$$

$$\underline{\beta}_{\tilde{W},i}\left(|(e_i, s_i)|\right) \leqslant \tilde{W}_i(\kappa_i, l_i, e_i, s_i) \leqslant \bar{\beta}_{\tilde{W},i}\left(|(e_i, s_i)|\right). \tag{24}$$

The construction of the functions and constants mentioned in Condition 1 depends on which local medium access protocols are employed and can be done systematically. More details on this construction are provided in Section VI-B, see also [2].

A. Stability Analysis

In order to guarantee UGAS or UGES, consider the following addition to Condition 1. For the sake of brevity, we sometimes omit the arguments of $\tilde{W}_i(\kappa_i, l_i, e_i, s_i)$.

Condition 2: For each $i \in \bar{N}$, there exist continuous functions $H_{l_i,i} : \mathbb{R}^{n_x} \times \mathbb{R}^{(n_v - n_{v,i})} \to \mathbb{R}$ and constants $L_{l_i,i} \geqslant 0$, for $l_i \in \{0,1\}$, such that for all $\kappa_i \in \mathbb{N}$, $l_i \in \{0,1\}$, $s_i \in \mathbb{R}^{n_{v,i}}$, $x \in \mathbb{R}^{n_x}$, and almost all $e_i \in \mathbb{R}^{n_{v,i}}$ it holds that

$$\left\langle \frac{\partial \tilde{W}_i}{\partial e_i}, g_i(x, e_i, 0) \right\rangle \leqslant L_{l_i,i}\tilde{W}_i + H_{l_i,i}(x, \bar{e}_i). \tag{25}$$

Furthermore, there exists a locally Lipschitz function $\tilde{V} : \mathbb{R}^{n_x} \to \mathbb{R}_{\geqslant 0}$, $\mathcal{K}_\infty$-functions $\underline{\beta}_{\tilde{V}}$ and $\bar{\beta}_{\tilde{V}}$, positive definite functions $\tilde{\rho} : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ and $\sigma_{l_i,i} : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$, positive semi-definite functions $\tilde{\varrho}_i : \mathbb{R}^{n_{v,i}} \to \mathbb{R}_{\geqslant 0}$ and constants $\gamma_{l_i,i} > 0$, $l_i \in \{0,1\}$, $i \in \bar{N}$, such that for all $\kappa \in \mathbb{N}^N$, $l \in \{0,1\}^N$, $s, e \in \mathbb{R}^{n_v}$, and almost all $x \in \mathbb{R}^{n_x}$, it holds that

$$\begin{aligned}&\left\langle \nabla \tilde{V}(x), f(x,e,0) \right\rangle \leqslant -\tilde{\rho}\left(|x|\right) \\ &\quad + \sum_{i=1}^{N} \left(-\tilde{\varrho}_i(v_i) - H_{l_i,i}^2(x,e) - \sigma_{l_i,i}(\tilde{W}_i) + \gamma_{l_i,i}^2 \tilde{W}_i^2 \right)\end{aligned} \tag{26}$$

and for all $x \in \mathbb{R}^{n_x}$

$$\underline{\beta}_{\tilde{V}}\left(|x|\right) \leqslant \tilde{V}(x) \leqslant \bar{\beta}_{\tilde{V}}\left(|x|\right). \tag{27}$$

The condition (26) is essentially a scaled $\mathcal{L}_2$-gain condition from $(\tilde{W}_1, \tilde{W}_2, \ldots, \tilde{W}_N)$ to $(H_{l_1,1}, H_{l_2,2}, \ldots, H_{l_N,N})$, which in the special case of a centralized control configuration, i.e., when taking $N = 1$, is similar to the conditions used in [1], [2], and [6]. In fact, the inequalities (25) and (26) recover the conditions used in [2] for $N = 1$ and form a generalization for the decentralized multi-network case $(N > 1)$. Consider now functions $\phi_{l_i,i} : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$, with $i \in \bar{N}$ and $l_i \in \{0,1\}$, which evolve according to

$$\frac{d}{d\tau_i}\phi_{l_i,i} = \begin{cases} -2L_{l_i,i}\phi_{l_i,i} - \gamma_{l_i,i}(\phi_{l_i,i}^2 + 1), & \text{for } \tau_i \in \left[0, \tau_{\text{miet}}^i\right] \\ 0, & \text{for } \tau_i > \tau_{\text{miet}}^i \end{cases} \tag{28}$$

where $L_{l_i,i}$ and $\gamma_{l_i,i}$ are constants given in Condition 2 and the initial conditions $\phi_{l_i,i}(0)$ are still to be specified.

Theorem 1: Consider the system $\mathcal{H}$ that satisfies Condition 1 and Condition 2. Suppose the following statements hold for each $i \in \bar{N}$.

(i) There exist positive real constants τ_{miet}^i and τ_{mad}^i with $\tau_{\text{miet}}^i \geqslant \tau_{\text{mad}}^i$ satisfying

$$\gamma_{0,i}\phi_{0,i}(\tau_{\text{miet}}^i) \geqslant \lambda_i^2 \gamma_{1,i}\phi_{1,i}(0) \tag{29}$$

$$\gamma_{1,i}\phi_{1,i}(\tau_i) \geqslant \gamma_{0,i}\phi_{0,i}(\tau_i), \text{ for all } \tau_i \in \left[0, \tau_{\text{mad}}^i\right] \tag{30}$$

where $\phi_{0,i}$ and $\phi_{1,i}$ evolve according to (28) for some fixed initial conditions $\phi_{l_i,i}(0)$ that satisfy $\gamma_{1,i}\phi_{1,i}(0) \geqslant \gamma_{0,i}\phi_{0,i}(0) > \lambda_i^2\gamma_{1,i}\phi_{1,i}(0) > 0$, and with λ_i as in (22).

(ii) There exist a function $\Psi_i : \mathbb{R}^{n_{v,i}} \times \mathbb{R}^{n_{v,i}} \times \mathbb{R}_{\geqslant 0} \times \mathbb{N} \times \mathbb{R}^{n_{v,i}} \times \{0,1\} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ with $\Psi_i(\cdot,\cdot,\cdot,\cdot,\cdot,\cdot,\eta_i)$ locally Lipschitz for all $\eta_i \in \mathbb{R}_{\geqslant 0}$ and a locally Lipschitz $\mathcal{K}_\infty$ function $\delta_{\eta_i} : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$, such that for all $\xi \in \mathbb{X}$

$$\Psi_i(o_i) \leqslant M_i(\xi) - \delta_{\eta_i}\left(|\eta_i|\right) \tag{31}$$

and for all $\xi \in \mathbb{X}$ with $0 \leqslant \tau_i \leqslant \tau_{\text{miet}}^i$

$$\Psi_i(o_i) + \delta_{\eta_i}(\eta_i) \geqslant 0 \tag{32}$$

where $o_i = (v_i, e_i, \tau_i, \kappa_i, s_i, l_i, \eta_i)$ and where

$$M_i(\xi) = \begin{cases} M_{1,i}(\xi), & \text{for } \tau_i \in \left[0, \tau_{\text{miet}}^i\right] \\ M_{2,i}(\xi), & \text{for } \tau_i > \tau_i > \tau_{\text{miet}}^i \end{cases} \tag{33}$$

with

$$M_{1,i}(\xi) := \tilde{\varrho}_i(v_i) + \left(H_{l_i,i}(x,\bar{e}_i) - \gamma_{l_i,i}\phi_{l_i,i}\tilde{W}_i\right)^2 \tag{34}$$

$$\begin{aligned}M_{2,i}(\xi) &:= \tilde{\varrho}_i(v_i) + H_{0,i}^2(x,\bar{e}_i) - 2\gamma_{0,i}\phi_{0,i}\tilde{W}_i H_{0,i}(x,\bar{e}_i) \\ &\quad - \left(\gamma_{0,i}^2 + 2\gamma_{0,i}\phi_{0,i}L_{0,i}\right)\tilde{W}_i^2.\end{aligned} \tag{35}$$

Then the event generator given by (6) and (7) assures that for the hybrid system $\mathcal{H}$, described by (1) and (10)–(18), the set $\mathcal{E}$, as defined in Definition 4, is UGAS. If, in addition, for each $i \in \bar{N}$ there exist strictly positive numbers $b_{1,i}$, $b_{2,i}$, c_1, c_2, $c_{4,i}$ and $c_{5,i}$ such that for all $r \in \mathbb{R}_{\geqslant 0}$, $\underline{\beta}_{\tilde{W},i}(r) = b_{1,i}r$, $\bar{\beta}_{\tilde{W},i}(r) = b_{2,i}r$, $\underline{\beta}_{\tilde{V}}(r) = c_1 r^2$, $\bar{\beta}_{\tilde{V}}(r) = c_2 r^2$, $\tilde{\rho}(r) \geqslant c_3 r^2$, $\delta_{\eta_i}(r) \geqslant c_{4,i} r$, and $\sigma_{l_i}(r) \geqslant c_{5,i} r^2$, $l_i = 0,1$, then the set $\mathcal{E}$ is UGES.

The proof is given in the Appendix. Observe that the functions Ψ_i only depend on the local (state) variables $o_i \in \mathbb{O}_i$ and not on the entire state ξ according to (31). This is needed to warrant output-based decentralized implementation of the event generators.

B. $\mathcal{L}_p$-Gain Performance Analysis

A convenient way to analyze the $\mathcal{L}_p$-gain of a control system is by constructing a so-called *storage function* S, which is positive semi-definite, and satisfies the dissipation inequality $\dot{S} \leqslant \theta^p |w|^p - |q(x,w)|^p$ during flow and $S^+ \leqslant S$ at jumps, where $\theta^p|w|^p - |q(x,w)|^p$ is the supply rate, θ an upper bound for the $\mathcal{L}_p$-gain and $q(x,w)$ as in (20) [43]. In order to construct such a storage function, we first consider the following conditions.

Condition 3 [2], [6]: For each $i \in \bar{N}$, there exist continuous functions $H_{l_i,i} : \mathbb{R}^{n_x} \times \mathbb{R}^{(n_v - n_{v,i})} \times \mathbb{R}^{n_w} \to \mathbb{R}$ and constants $L_{l_i,i} \geqslant 0$, for $l_i \in \{0,1\}$, such that for all $\kappa_i \in \mathbb{N}$, $l_i \in \{0,1\}$, $s_i \in \mathbb{R}^{n_{v,i}}$, $x \in \mathbb{R}^{n_x}$, and almost all $e_i \in \mathbb{R}^{n_{v,i}}$ it holds that

$$\left\langle \frac{\partial \tilde{W}_i}{\partial e_i}, g_i(x, e_i, w) \right\rangle \leqslant L_{l_i,i}\tilde{W}_i + H_{l_i,i}(x, \bar{e}_i, w). \tag{36}$$

Furthermore, there exist a locally Lipschitz function $\tilde{V} : \mathbb{R}^{n_x} \to \mathbb{R}_{\geqslant 0}$, $\mathcal{K}_\infty$-functions $\underline{\beta}_{\tilde{V}}$ and $\bar{\beta}_{\tilde{V}}$, positive semi-definite functions $\tilde{\varrho}_i : \mathbb{R}^{n_{v,i}} \to \mathbb{R}_{\geqslant 0}$, $i \in \bar{N}$, and constants $\alpha_i, \gamma_{l_i,i} > 0$, $l_i \in \{0,1\}$, $i \in \bar{N}$, such that for all $\kappa \in \mathbb{N}$, $s, e \in \mathbb{R}^{n_v}$, $w \in \mathbb{R}^{n_w}$, $l \in \{0,1\}^N$, and almost all $x \in \mathbb{R}^{n_x}$, it holds that

$$\left\langle \nabla \tilde{V}(x), f(x,e,w) \right\rangle \leqslant \sum_{i=1}^{N} \Big(-\tilde{\varrho}_i(v_i) - H^2_{l_i,i}(x, \bar{e}_i, w) + \gamma^2_{l_i,i} \tilde{W}_i^2 \Big) + \tilde{\mu} \left(\theta^p |w|^p - |q(x,w)|^p \right) \quad (37)$$

for some constants $\tilde{\mu} > 0$ and $\theta \geqslant 0$, and for all $x \in \mathbb{R}^{n_x}$ it holds that

$$\underline{\beta}_{\tilde{V}}(|x|) \leqslant \tilde{V}(x) \leqslant \bar{\beta}_{\tilde{V}}(|x|). \quad (38)$$

Theorem 2: Consider the system $\mathcal{H}$ that satisfies Condition 1, Condition 3, and condition (i) of Theorem 1. Suppose there exists a function $\Psi_i : \mathbb{R}^{n_{v,i}} \times \mathbb{R}^{n_{v,i}} \times \mathbb{R}_{\geqslant 0} \times \mathbb{N} \times \mathbb{R}^{n_{v,i}} \times \{0,1\} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ with $\Psi_i(\cdot,\cdot,\cdot,\cdot,\cdot,\cdot,\eta_i)$ locally Lipschitz for all $\eta_i \in \mathbb{R}_{\geqslant 0}$ for each $i \in \bar{N}$ that satisfies for all $\xi \in \mathbb{X}$

$$\Psi_i(o_i) \leqslant \ M_i(\xi, w) \quad (39)$$

$$\Psi_i(o_i) \geqslant \ 0, \text{ when } 0 \leqslant \tau_i \leqslant \tau^i_{\text{miet}} \quad (40)$$

where $o_i = (v_i, e_i, \tau_i, \kappa_i, s_i, l_i, \eta_i)$ and where

$$M_i(\xi, w) = \begin{cases} M_{1,i}(\xi, w), & \text{for } 0 \leqslant \tau_i \leqslant \tau^i_{\text{miet}} \\ M_{2,i}(\xi, w), & \text{for } \tau_i > \tau^i_{\text{miet}} \end{cases}$$

with

$$M_{1,i}(\xi, w) := \ \tilde{\varrho}_i(v_i) + \Big(H_{l_i,i}(x, \bar{e}_i, w) - \gamma_{l_i,i} \phi_{l_i,i} \tilde{W}_i \Big)^2 \quad (41)$$

$$M_{2,i}(\xi, w) := \ \tilde{\varrho}_i(v_i) - 2\gamma_{0,i}\phi_{0,i}\tilde{W}_i H_{0,i}(x, \bar{e}_i, w) + H^2_{0,i}(x, \bar{e}_i, w) - \left(\gamma^2_{0,i} + 2\gamma_{0,i}\phi_{0,i}L_{0,i} \right) \tilde{W}_i^2. \quad (42)$$

Then the event-generator given by (6) and (7) guarantees that the system $\mathcal{H}$, described by (1) and (10)–(18), is $\mathcal{L}_p$-stable with respect to input w and output z according to (20) with an $\mathcal{L}_p$-gain less than or equal to θ.

The proof is given in the Appendix.

Remark 3: The conditions presented in this section can also be used to determine time-based stability and $\mathcal{L}_p$-gain conditions for decentralized (multiple-networks) time-triggered NCSs in terms of τ^i_{mati} and τ^i_{mad}. Indeed, by taking $\tilde{\varrho}_i(v_i) = 0$, for all $i \in \bar{N}$, and $\tau^i_{\text{mati}} = \tau^i_{\text{miet}}$, where τ^i_{miet} and τ^i_{mad} follow from (29) and (30), respectively, for each $\mathcal{N}_i$ $(\tau^i_{\text{mati}}, \tau^i_{\text{mad}})$-conditions are obtained guaranteeing UGAS/UGES of $\mathcal{E}$ and an $\mathcal{L}_p$-gain less than or equal to θ. If we consider the centralized (single-network) time-triggered case, i.e., $N = 1$, we recover the results presented in [2].

VI. Design of Event Generators

In this section we will demonstrate how to construct the event generator as in (6) and (7) through the definition of the functions $\Psi_i : \mathbb{O}_i \to \mathbb{R}$, $i \in \bar{N}$, satisfying (31) and (32) in case we can assume that $w = 0$ and the control objective is to guarantee UGAS of $\mathcal{E}$, or satisfying (39) and (40) in case $w \neq 0$ and the control objective is to guarantee $\mathcal{L}_p$ stability.

A. General Design Considerations

As already mentioned, due to the decentralized control setup, only local information $o_i \in \mathbb{O}_i$ is available at each ETM. Hence, the functions Ψ_i, $i \in \bar{N}$, have to be constructed such that they only depend on locally available variables. To do so, we first need to derive lower-bounds for the functions $M_{1,i}$ and $M_{2,i}$, as given in (34) and (35), respectively, which only depend on these local variables. A lower bound for the function $M_{1,i}$ can easily be obtained from (34), namely

$$M_{1,i}(\xi) \geqslant \tilde{\varrho}_i(v_i). \quad (43)$$

By employing Lemma 1, we can derive a lower bound for $M_{2,i}$ in (35) as

$$\begin{aligned} M_{2,i} &\geqslant \tilde{\varrho}_i + H^2_{0,i} - H^2_{0,i} - (\gamma_{0,i}\phi_{0,i}\tilde{W}_i)^2 \\ &\quad - \left(\gamma^2_{0,i} + 2\gamma_{0,i}\phi_{0,i}L_{0,i} \right) \tilde{W}_i^2 \\ &\geqslant \tilde{\varrho}_i - \gamma_{0,i} \left(2\phi_{0,i}L_{0,i} + \gamma_{0,i}\left(1 + \phi^2_{0,i}\right)\right) \tilde{W}_i^2. \end{aligned} \quad (44)$$

By combining (32), (43), and (44), we can now define the functions Ψ_i, $i \in \bar{N}$, which satisfy (31), as

$$\Psi_i(o_i) = \tilde{\varrho}_i(v_i) - \delta_{\eta_i}(\eta_i) - (1 - \omega_i(\tau_i))\, \bar{\gamma} \tilde{W}_i^2 \quad (45)$$

where

$$\omega_i(\tau_i) := \begin{cases} 1, & \text{for } 0 \leqslant \tau_i \leqslant \tau^i_{\text{miet}} \\ 0, & \text{for } \tau_i > \tau^i_{\text{miet}} \end{cases} \quad (46)$$

and where $\bar{\gamma}_i = \gamma_{0,i}(2\phi_{0,i}L_{0,i} + \gamma_{0,i}(1 + \phi^2_{0,i}))$, which is, given (28), a constant for $\tau_i \geqslant \tau^i_{\text{miet}}$. Due to (45) and (46), the flow map F as given in (10) is discontinuous in τ_i. However, since $\dot{\tau}_i = 1$, this does not cause any problems in the existence of solutions.

For the $\mathcal{L}_p$-stability case, we can define Ψ_i similar to (45) by taking $\delta_{\eta_i}(\eta_i) = 0$.

Remark 4: The functions $\tilde{\varrho}_i, i \in \bar{N}$, can be arbitrary semi-positive definite functions. From (31), (34), and (35) for the UGAS case and from (39), (41), and (42) for the $\mathcal{L}_p$-stability case, we can see that if $\tilde{\varrho}_i, i \in \bar{N}$, is chosen positive definite, the bound on Ψ_i becomes less stringent than the case where $\tilde{\varrho}_i = 0$, $i \in \bar{N}$, in the sense that Ψ_i can be chosen larger with respect to the arguments u_i and y_i. From (7), we can see that if Ψ_i is larger, it will in general take longer before η_i in (6) becomes non-positive. The drawback is that the bound on the derivative of V given by (26) becomes more stringent as well. As a consequence, $\gamma_{l_i,i}$, for $i \in \bar{N}$ and $l_i \in \{0,1\}$, have to increase which implies that τ^i_{miet} and τ^i_{mad} will decrease. Note, however, that only for the situation that y_i and u_i are relatively small for $0 \leqslant \tau_i \leqslant \tau^i_{\text{miet}}$ and $\Psi_i < 0$ for $\tau_i > \tau^i_{\text{miet}}$, inter-event times close to τ^i_{miet} can be expected. Typically, this situation corresponds to the case that the disturbance is close to zero for $0 \leqslant \tau_i \leqslant \tau^i_{\text{miet}}$ and is large for $\tau_i > \tau^i_{\text{miet}}$. Due to this dependency on τ_i, this worst-case disturbance is unlikely to occur in a practical setting.

Thus, by choosing $\tilde{\varrho}_i$ larger, we can derive a triggering condition which in general yields larger inter-event times on average (up to a certain point). Hence, this discussion already reveals that there is a trade-off between the guaranteed minimum inter-event times, robustness in terms of MADs and the expected average inter-event times.

Remark 5: The conditions given by (32) and (40) assure that $\eta_i(t) \geqslant 0$ when $0 \leqslant \tau_i \leqslant \tau^i_{\text{miet}}$. Hence, the dynamic triggering mechanism given by (6) and (7) can be modified to a static triggering mechanism as follows:

$$t^i_0 = 0,\ t^i_{j+1} := \inf\left\{t \geqslant t^i_j + \tau^i_{\text{miet}} | \Psi_i(o_i) \leqslant 0\right\} \tag{47}$$

where Ψ_i as in (45) with $\delta_{\eta_i}(\eta_i) = 0$. Observe that according to this triggering condition, a transmission event takes place in network $\mathcal{N}_i$, each time $\eta_i(t)$ starts decreasing in contrast to the dynamic triggering mechanism given by (6) and (7). Hence, the dynamic triggering condition outperforms its static alternative in any case with respect to the inter-event times.

Remark 6: In the specific case where no delays and no disturbances are present, where only one network with one node is considered, and where we employ the static triggering condition given by (47), we recover the design in [27].

B. Construction of Lyapunov and Storage Functions

To construct the Lyapunov and storage functions $\tilde{V}$ and $\tilde{W}_i$, and the constants as presented in Condition 1–3, we are inspired by the results in [2] and [42]. The starting points are based on well known conditions, as used in [1], [4], and [6] for the delay-free case.

Condition 4: All protocols are UGES, in the sense that for each $i \in \bar{N}$ there exists a function $W_i : \mathbb{N} \times \mathbb{R}^{n_{v,i}} \to \mathbb{R}_{\geqslant 0}$ that is locally Lipschitz in its second argument such that for all $\kappa_i \in \mathbb{N}$ and all $e_i \in \mathbb{R}^{n_{v,i}}$ it holds that

$$\underline{\alpha}_{W,i}|e_i| \leqslant W_i(\kappa_i, e_i) \leqslant \bar{\alpha}_{W,i}|e_i| \tag{48a}$$

$$W_i\left(\kappa_i + 1, h_i(\kappa_i, e_i)\right) \leqslant \lambda_i W_i(\kappa_i, e_i) \tag{48b}$$

for constants $0 < \underline{\alpha}_{W,i} \leqslant \bar{\alpha}_{W,i}$ and $0 < \lambda_i < 1$.

For the standard sampled-data (SD) systems where $h_i(\kappa_i, e_i) = 0$ for all $\kappa_i \in \mathbb{N}, e_i \in \mathbb{R}^{n_{v,i}}$, we can take $W_i(\kappa_i, e_i) = |e_i|$ and $\lambda_i > 0$ arbitrary small. The TOD protocol, the round-robin protocol, and many more are also known to satisfy the above mentioned conditions, see [1] and [2] for details on how to construct the corresponding functions W_i. Additional to Condition 4, we require that for all $\kappa_i \in \mathbb{N}$, $e_i \in \mathbb{R}^{n_{v,i}}$

$$W_i(\kappa_i + 1, e_i) \leqslant \lambda_{W,i} W_i(\kappa_i, e_i) \tag{49}$$

for some constant $\lambda_{W,i} \geqslant 1$, and that for almost all $e_i \in \mathbb{R}^{n_{v,i}}$ and all $\kappa_i \in \mathbb{N}$

$$\left|\frac{\partial W_i}{\partial e_i}(\kappa_i, e_i)\right| \leqslant c_i \tag{50}$$

for some constant $c_i \in \mathbb{R}_{>0}$. Moreover, in case $w = 0$ and thus if we aim to find a Lyapunov function to guarantee UGAS (and sometimes even UGES), we assume that the growth of e_i is bounded according to

$$|g_i(x, e_i, 0)| \leqslant c_i^{-1}\left(H_{x,i}(x, \bar{e}_i) + M_{e,i}|e_i|\right) \tag{51}$$

where $H_{x,i} : \mathbb{R}^{n_{x,i}} \times \mathbb{R}^{(n_v - n_{v,i})} \to \mathbb{R}_{\geqslant 0}$, and, in case $w \neq 0$ and thus if we aim to find a storage function to guarantee $\mathcal{L}_p$-stability, we assume that

$$|g_i(x, e_i, w)| \leqslant c_i^{-1}\left(H_i(x, \bar{e}_i, w) + M_{e,i}|e_i|\right) \tag{52}$$

where $H_i : \mathbb{R}^{n_x} \times \mathbb{R}^{(n_v - n_{v,i})} \times \mathbb{R}^{n_w} \to \mathbb{R}_{\geqslant 0}$. In both cases $M_{e,i} \geqslant 0$ is a constant. Similar to Condition 2, we establish a scaled $\mathcal{L}_2$-gain from $(W_1, W_2, \ldots, W_N)$ to $(H_{x,1}, H_{x,2}, \ldots, H_{x,N})$, by assuming the existence of a locally Lipschitz continuous function $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geqslant 0}$, which is radially unbounded, i.e., for all $x \in \mathbb{R}^{n_x}$

$$\underline{\alpha}_V\left(|x|\right) \leqslant V(x) \leqslant \bar{\alpha}_V\left(|x|\right) \tag{53}$$

for some $\mathcal{K}_\infty$-functions $\underline{\alpha}_V$ and $\bar{\alpha}_V$, and

$$\begin{aligned}\langle \nabla V(x), f(x, e, 0)\rangle \leqslant& -\rho\left(|x|\right) - \sum_{i=1}^{N}\left(H^2_{x,i}(x, \bar{e}_i) + \varrho_i(v_i)\right)\\ &+ \sum_{i=1}^{N}\left(\gamma_i^2 - \varepsilon\right) W_i^2(\kappa_i, e_i)\end{aligned} \tag{54}$$

for almost all $x \in \mathbb{R}^{n_x}$ and all $e \in \mathbb{R}^{n_v}$ with $\rho : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ positive definite and some semi-positive definite functions $\varrho_i : \mathbb{R}^{n_{v,i}} \to \mathbb{R}_{\geqslant 0}$ and where the constants ε, γ_i satisfy $0 < \varepsilon < \min\{\gamma_i^2, 1\}$, where ε is typically taken small. A similar condition can be obtained to construct a storage function V for $\mathcal{L}_p$-stability, namely

$$\begin{aligned}\langle \nabla V(x), f(x, e, w)\rangle \leqslant& -\sum_{i=1}^{N}\left(H_i^2(x, \bar{e}_i, w) + \varrho_i(v_i)\right)\\ &+ \sum_{i=1}^{N}\gamma_i^2 W_i^2(\kappa_i, e_i) + \mu\left(\theta^p|w|^p - |q(x, w)|^p\right)\end{aligned} \tag{55}$$

for almost all $x \in \mathbb{R}^{n_x}$, all $e \in \mathbb{R}^{n_v}$, and all $w \in \mathbb{R}^{n_w}$.

Based on the above mentioned conditions, which correspond to the delay-free case, see [2], the constants and functions satisfying Conditions 1–3, which are imposed in case variable transmission delays are present, can now be obtained.

Proposition 3: Consider system $\mathcal{H}$. If Condition 4 and inequalities (49)–(51), (53), and (54) hold (in case $w = 0$) or Condition 4 and inequalities (49), (50), (52), (53), and (55) hold (in case $w \neq 0$), then the functions $\tilde{V}$ and $\tilde{W}_i$ given by

$$\tilde{W}_i(\kappa_i, 0, e_i, s_i) = \max\left\{W_i(\kappa_i, e_i), W_i(\kappa_i, e_i + s_i)\right\}$$

$$\tilde{W}_i(\kappa_i, 1, e_i, s_i) = \max\left\{\frac{\lambda_i}{\lambda_{W,i}} W_i(\kappa_i, e_i), W_i(\kappa_i, e_i + s_i)\right\}$$

$$\tilde{V}(x) = V(x)$$

satisfy Condition 1 and Condition 2, and Condition 1 and Condition 3, respectively, with $\underline{\beta}_{\tilde{W},i}(r) = \underline{\beta}_{\tilde{W},i}r$, $\bar{\beta}_{\tilde{W},i}(r) = \bar{\beta}_{\tilde{W},i}r$, $\underline{\beta}_{\tilde{V}} = \underline{\alpha}_V$, $\bar{\beta}_{\tilde{V}} = \bar{\alpha}_V$, $\sigma_{0,i}(r) = \varepsilon r^2$, $\sigma_{1,i}(r) = \varepsilon \lambda_{W,i}^2/\lambda_i^2 r^2$, $\tilde{\rho}(r) = \rho(r)$, $\tilde{\varrho}_i(y_i, u_i) = \varrho(y_i, u_i)$, $H_{l_i,i}(x, \bar{e}_i) = H_{x,i}(x, \bar{e}_i)$, $H_{l_i,i}(x, \bar{e}_i, w) = H_i(x, \bar{e}_i, w)$, $\tilde{\mu} = \mu$, and

$$L_{0,i} = \frac{M_{e,i}}{\underline{\alpha}_{W,i}}, \quad L_{1,i} = \frac{M_{e,i}\lambda_{W,i}}{\lambda_i \underline{\alpha}_{W,i}} \tag{56}$$

$$\gamma_{0,i} = \gamma_i, \qquad \gamma_{1,i} = \frac{\gamma_i \lambda_{W,i}}{\lambda_i} \tag{57}$$

where $\underline{\beta}_{\tilde{W},i}$ and $\bar{\beta}_{\tilde{W},i}$.

The proof is based on [2], and is omitted here for space reasons.

The following lemma can be used to implement (45) in practice, where the mode l_i might not be available to the event generator, i.e., in case the delays are unknown the update times (and thus the modes l_i) are also unknown.

Lemma 2: Consider system $\mathcal{H}$ and suppose there exists a function W_i that satisfies Condition 4, (49) and (50), then

$$(1 - \omega_i(\tau_i))\, \bar{\gamma} \tilde{W}_i^2 = (1 - \omega_i(\tau_i))\, \bar{\gamma}_i W_i^2 \tag{58}$$

with $\tilde{W}_i$ as given in Proposition 3 and ω_i as given in (46).

Proof: According to (46) we have that $1 - \omega_i(\tau_i) = 0$ for $0 \leqslant \tau_i \leqslant \tau_{\text{miet}}^i$. Hence, $(1-\omega_i(\tau_i))\bar{\gamma}_i \tilde{W}_i^2 = (1-\omega_i(\tau_i))\bar{\gamma}_i W_i^2 = 0$ when $0 \leqslant \tau_i \leqslant \tau_{\text{miet}}^i$. Furthermore, given the fact that $\tau_{\text{mad}}^i \leqslant \tau_{\text{miet}}^i$ due to Standing Assumption 1, then for $\tau_i > \tau_{\text{miet}}$, the next event in network $\mathcal{N}_i$ is a transmission event, i.e., $l_i = 0$. Since, $s_i = 0$ for $l_i = 0$ when $\tau_i > \tau_{\text{miet}}^i$ according to (10) and (15), we can see that $\tilde{W}_i = \max\{W_i(\kappa_i, e_i), W_i(\kappa_i, e_i + s_i)\} = W_i$ for $\tau_i > \tau_{\text{miet}}^i$, which completes the proof. ■

Remark 7: Note that the conditions presented above put some restrictions on the class of nonlinear systems that can be considered. Indeed, observe that the inequalities (51) and (52) require that $g_i(x, e_i, w)$ is linearly bounded with respect to e_i. In case the nonlinear terms in $f_i(x, e_i, w)$ and $g_i(x, e_i, w)$ are linearly bounded with respect to all arguments (e.g., in case f_i and g_i are globally Lipschitz maps), one can essentially apply the results for linear systems presented in Section VI-C mutatis mutandis. However, the presented framework is not restricted to this class of linearly bounded systems as we demonstrate by an example in Section VII-B.

C. The Linear Case

In this section, we will discuss how to systematically construct the functions and constants satisfying the conditions presented in Section VI-B, when $f(x, e, w)$ as in (10) can be written as

$$\dot{x} = A_{11}x + A_{12}e + A_{13}w \tag{59}$$

with $v_i = C_i x$ and where the performance output (20) is given by

$$z = C_z x + D_z w. \tag{60}$$

Based on (59), we find that the dynamics of the transmission errors $e_i, i \in \bar{N}$, are given by

$$\dot{e}_i = A_{21,i}x + A_{22,i}e + A_{23,i}w \tag{61}$$

where $A_{21,i} = -C_i A_{11}$, $A_{22,i} = -C_i A_{12}$, and $A_{23,i} = -C_i A_{13}$, $i \in \bar{N}$. By employing the notation

$$A_{21} = \begin{bmatrix} A_{21,1} \\ \vdots \\ A_{21,N} \end{bmatrix}, \; A_{22} = \begin{bmatrix} A_{22,1} \\ \vdots \\ A_{22,N} \end{bmatrix}, \; A_{23} = \begin{bmatrix} A_{23,1} \\ \vdots \\ A_{23,N} \end{bmatrix}$$

we can write the dynamics of the transmission errors compactly as

$$\dot{e} = A_{21}x + A_{22}e + A_{23}w. \tag{62}$$

For the sake of brevity, we will only consider the $\mathcal{L}_2$-gain analysis.

In order to construct the functions Ψ_i for each network $\mathcal{N}_i$, $i \in \bar{N}$, that satisfy (39) and (40), we first have to find the functions W_i and the constants c_i, λ_i, $\lambda_{W,i}$, $\underline{\alpha}_{W,i}$, and $\bar{\alpha}_{W,i}$ such that Condition 4 and inequalities (49) and (50) hold. For the SD protocol, we can take $W_{i,\text{SD}}(\kappa_i, e_i) = |e_i|$, $\underline{\alpha}_{W,i,\text{SD}} = \bar{\alpha}_{W,i,\text{SD}} = 1$, $c_{i,\text{SD}} = 1$, and $\lambda_{W,i,\text{SD}} = 1$ in (48)–(50). Furthermore, $0 < \lambda_{i,\text{SD}} < 1$, can be chosen arbitrarily. See [1] for more details on how to construct the functions W_i and the constants c_i, λ_i, $\lambda_{W,i}$, $\underline{\alpha}_{W,i}$, and $\bar{\alpha}_{W,i}$ for other protocols including the RR and TOD protocol.

To construct a function V satisfying (55), we introduce $\Gamma := \text{diag}(\gamma_1 I_{n_{e,1}}, \gamma_2 I_{n_{e,2}}, \ldots, \gamma_N I_{n_{e,N}})$, to obtain

$$\sum_{i=1}^{N} \gamma_i^2 W_{i,\text{SD}}^2(\kappa_i, e_i) = e^\top \Gamma^2 e. \tag{63}$$

In order to find the function $H_i(x, \bar{e}_i, w)$ (as needed in (55)) and constant $M_{e,i}$ satisfying (52), consider that (61) implies

$$|\dot{e}_i| \leqslant |A_{21,i}x + \tilde{A}_{22,i}e + A_{23,i}w| + |A_{22,i}\bar{\Gamma}_i e| \tag{64}$$

where $\tilde{A}_{22,i} := A_{22,i}(I_{n_v} - \bar{\Gamma}_i)$. Note that the term $\tilde{A}_{22,i}e$ is independent of e_i.

Based on (64), (52) can be satisfied by defining $M_{e,i}$ and $H_i(x, \bar{e}_i, w)$ as

$$M_{e,i} := c_i \sqrt{\left(\lambda_{\max}(\bar{\Gamma}_i A_{22,i}^\top A_{22,i} \bar{\Gamma}_i)\right)} \tag{65}$$

$$H_i(x, \bar{e}_i, w) := c_i |A_{21,i}x + \tilde{A}_{22,i}e + A_{23,i}w| \tag{66}$$

where c_i is a constant satisfying (50), as just specified for the SD protocol. By introducing $\bar{C} := \text{diag}(c_1 I_{n_{e,1}}, c_2 I_{n_{e,2}}, \ldots, c_N I_{n_{e,N}})$, we obtain

$$\sum_{i=1}^{N} H_i^2(x, w) = \left| \bar{C}(A_{21}x + \tilde{A}_{22}e + A_{23}w) \right|^2. \tag{67}$$

As already mentioned, the function ϱ_i in (55) is an arbitrary positive semi-definite function. Hence, we can take ϱ_i in the quadratic form

$$\varrho_i(v_i) = v_i^\top Q_i v_i \tag{68}$$

where Q_i is an arbitrary positive semi-definite matrix. Since $v_i = C_i x$, we have that

$$\sum_{i=1}^{N} \rho_i(v_i) = x^\top C^\top Q C x \tag{69}$$

where $C := [C_1^\top, C_2^\top, \ldots, C_N^\top]^\top$ and $Q := \mathrm{diag}(Q_1, Q_2, \ldots, Q_N)$.

Now consider the quadratic candidate storage function $V(x) = x^T P x$. By using (59), (60), (63), (67), and (69), the inequality given in (55) can be formulated in terms of the linear matrix inequality (LMI) given by (70), shown at the bottom of the page. From (28) and (57), we can conclude that smaller values of γ_i yield, in general, larger minimum inter-event times. Hence, matrix P and constant μ can be computed by minimizing a weighted sum $\sum_{i=1}^{N} \alpha_i \gamma_i^2$, where $\alpha_i \in \mathbb{R}_{>0}$, $i \in \bar{N}$ subject to the LMI given in (70) for a fixed $\mathcal{L}_2$-gain θ. Note that different combinations of $(\alpha_1, \alpha_2, \ldots, \alpha_N)$ lead to different $(\gamma_1, \gamma_2, \ldots, \gamma_N)$ and thus different tradeoffs in resource utilization among different networks.

Hence, following the above procedure with the LMI (70) satisfied, gives in view of (45) and Lemma 2, that the hybrid system $\mathcal{H}$ has an $\mathcal{L}_2$-gain smaller than or equal to θ, if the functions Ψ_i in the ETMs as described by (6) and (7) are selected as

$$\Psi_i(o_i) = \begin{bmatrix} y_i \\ u_i \end{bmatrix}^\top Q_i \begin{bmatrix} y_i \\ u_i \end{bmatrix} - (1 - \omega_i(\tau_i))\, \bar{\gamma}_i W_i^2 \tag{71}$$

with $\omega_i(\tau_i)$ as in (46) and $\bar{\gamma}_i$ as in (45).

VII. Numerical Example

In this section we illustrate the use of the above analysis in synthesizing the ETMs as described by (6) and (7) for decentralized and output-based NCSs. First, a linear example is presented, which reveals tradeoffs between robustness (in terms of MADs), performance (in terms of the $\mathcal{L}_2$-gain of the overall system) and network utilization (in terms of MIETs and average inter-event times). Second, we consider a nonlinear example. For both examples, we compare the dynamic event-triggering condition (6) with the more commonly used static event-triggering condition (47). Both examples employ the sampled-data protocol for the local networks. For a numerical example considering the RR and TOD protocol, see [37]. The simulation results are obtained using the Hybrid Systems Simulation Toolbox [44] in Matlab/Simulink.

A. Linear Example

We study in the first example the problem of stabilizing two coupled cart-pendulum systems $\mathcal{P}_i, i \in \{1, 2\}$, with the pendula in their (unstable) upright equilibria. Each subsystem consists of a moving support (cart) with mass M_i, a rigid massless beam of length l_i, and a point mass m_i attached to the end of the beam, $i \in \{1, 2\}$. The end points of the pendula are coupled via a linear spring of stiffness k. The system is actuated via input forces u_i, $i \in \{1, 2\}$. Linearizing the pendula around their unstable upright equilibria, we find, with parameter values $M_1 = M_2 = 25$, $m_1 = m_2 = 5$, $l_1 = l_2 = 2$, $k = 0.1$, and gravitational acceleration $g = 10$,

$$\mathcal{P}_i : \begin{cases} \dot{x}_{p,i} &= A_{p,i,i} x_{p,i} + A_{p,i,j(i)} x_{p,j(i)} + B_{p,i} u_i + E_{p,i} w \\ y_i &= C_{p,i} x_{p,i} \end{cases}$$

for $i \in \{1, 2\}$, where $j(1) = 2$ and $j(2) = 1$ and where $x_{p,i}$ is the state of subsystem $\mathcal{P}_i$, $i \in \{1, 2\}$. The matrices $A_{p,i,j}$, $B_{p,i}$, $E_{p,i}$, and $C_{p,i}$, for $i, j \in \{1, 2\}$, are then given by

$$A_{p,1,1} = A_{p,2,2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2.9058 & 0 & -0.0054 & 0 \\ 0 & 0 & 0 & 1 \\ -1.6633 & 0 & 0.0017 & 0 \end{bmatrix}$$

$$A_{p,1,2} = A_{p,2,1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0108 & 0 & 0.0054 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0033 & 0 & -0.0017 & 0 \end{bmatrix}$$

$$B_{p,1} = B_{p,2} = \begin{bmatrix} 0 & -0.0042 & 0 & 0.0167 \end{bmatrix}^\top$$

$$E_{p,1} = B_{p,1}, \quad E_{p,2} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^\top$$

$$C_{p,1} = C_{p,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Each subsystem has its own, observer-based controller $\mathcal{C}_i$ collocated with the actuator (and thus $\hat{u}_i = u_i$, $i \in \{1, 2\}$), given by

$$\mathcal{C}_i : \begin{cases} \dot{x}_{c,i} = (A_{p,i,i} + B_{p,i} K_i) x_{c,i} + L_i (C_{p,i} x_{c,i} - \hat{y}_i) \\ u_i = K_i x_{c,i} \end{cases}$$

where $x_{c,i}$ is the state of controller $\mathcal{C}_i$ (and an estimate of $x_{p,i}$), and where K_i and L_i are such that the eigenvalues of $A_{K,1} := A_{p,1,1} + B_{p,1} K_1$ and $A_{L,1} := A_{p,1,1} + L_1 C_{p,1}$ are -1, -2, -3, -4, and the eigenvalues of $A_{K,2} := A_{p,2,2} + B_{p,2} K_2$ and $A_{L,2} := A_{p,2,2} + L_2 C_{p,2}$ are -2, -3, -4, -5. Particularly,

$$K_1 = \begin{bmatrix} 11321 & 7161.8 & 556.68 & 1190.4 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -5.998 & -10.902 & 0.052 & 1.785 \\ 0.063 & 0.164 & -4.002 & -3.006 \end{bmatrix}^\top$$

$$K_2 = \begin{bmatrix} 29071 & 18058 & 2833.3 & 3674.6 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -7.702 & -16.687 & 0.703 & 4.112 \\ 0.721 & 2.529 & -6.298 & -9.038 \end{bmatrix}^\top.$$

For each $i \in \{1, 2\}$, the output y_i is transmitted over network $\mathcal{N}_i$ using the sampled-data protocol (so $h_i = 0$ for $i = 1, 2$), leading to the control setup as shown in Fig. 2. By defining

$$\begin{pmatrix} A_{11}^\top P + P A_{11} + A_{21}^\top \bar{C}^2 A_{21} + \mu C_z^\top C_z + C^\top Q C & P A_{12} + A_{21}^\top \bar{C}^2 \tilde{A}_{22} & P A_{13} + A_{21}^\top \bar{C}^2 A_{23} + \mu C_z^\top D_z \\ A_{12}^\top P + \tilde{A}_{22}^\top \bar{C}^2 A_{21} & -\Gamma^2 + \tilde{A}_{22}^\top \bar{C}^2 \tilde{A}_{22} & \tilde{A}_{22}^\top \bar{C}^2 A_{23} \\ A_{13}^\top P + A_{23}^\top \bar{C}^2 A_{21} + \mu D_z^\top C_z & A_{23}^\top \bar{C}^2 \tilde{A}_{22} & \mu D_z^T D_z + A_{23}^\top \bar{C}^2 A_{23} - \mu \theta^2 I \end{pmatrix} \preceq 0, \; P = P^\top \succeq 0 \tag{70}$$

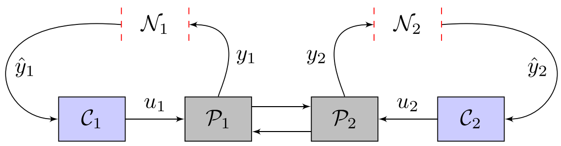

Fig. 2. Networked control setup of the two coupled cart-pendulum systems.

$x_i := (x_{p,i}, x_{c,i} - x_{p,i})$, the closed loops dynamics of (x_1, x_2) can be described by (59), where

$$A_{11} = \begin{bmatrix} A_{K,1} & B_{p,1}K_1 & A_{p,1,2} & 0 \\ 0 & A_{L,1} & -A_{p,1,2} & 0 \\ A_{p,2,1} & 0 & A_{K,2} & B_{p,2}K_2 \\ -A_{p,2,1} & 0 & 0 & A_{L,2} \end{bmatrix}$$

and where

$$A_{12} = \mathrm{diag}\left(\begin{bmatrix} 0 \\ -L_1 \end{bmatrix}, \begin{bmatrix} 0 \\ -L_2 \end{bmatrix}\right)$$
$$A_{13} = \begin{bmatrix} E_{p,1}^\top & -E_{p,1}^\top & E_{p,2}^\top & -E_{p,2}^\top \end{bmatrix}^\top$$

and by defining $e_i := \hat{y}_i - y_i$, the closed loop dynamics of e_i is given by (61) with

$$C_1 = -[C_{p,1} \quad 0 \quad 0 \quad 0]$$
$$C_2 = -[0 \quad 0 \quad C_{p,2} \quad 0].$$

The performance output is chosen as $z = C_z x$, where $x = (x_1, x_2)$ and $C_z = (\mathbf{0}_{10}^\top, 1, \mathbf{0}_5^\top)$. By using the results in Section VI-C, we have that $W_i = |e_i|$, $c_i = 1$, $0 < \lambda_i < 1$, $\lambda_{W,i} = 1$, $\underline{\alpha}_{W,i} = \bar{\alpha}_{W,i} = 1$, $H_i = |A_{21,i}x + A_{23,i}w|$, and $M_{e,i} = \sqrt{\lambda_{\max}(A_{22,i}^\top A_{22,i})}, i \in 1, 2$. Furthermore, we choose ϱ_i according to (68) with

$$Q_1 = 0.5\begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}, \; Q_2 = 0.3\begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}. \tag{72}$$

Now, we can minimize γ_i, $i = 1, 2$, subject to (70) for various values of θ and find τ_{miet}^i and τ_{mad}^i tradeoff curves by solving (29) and (30), respectively, for various λ_i, $\phi_{0,i}(0)$ and $\phi_{1,i}(0)$ that satisfy $\gamma_{1,i}\phi_{1,i}(0) \geqslant \gamma_{0,i}\phi_{0,i}(0) > \lambda_i^2\gamma_{1,i}\phi_{1,i}(0) > 0$, where $\gamma_{l_i,i}, l_i \in \{0,1\}, i \in \{1,2\}$ follow from (57). The corresponding dynamic event-triggering condition (6) and (7) and the corresponding static event-triggering condition (47) are constructed by taking Ψ_i as in (71).

In Fig. 3, τ_{miet}^i versus τ_{mad}^i tradeoff curves for both networks, corresponding to the case where an $\mathcal{L}_2$-gain of $\theta = 0.01$ is guaranteed, are shown. In this figure, also the τ_{mati}^i versus τ_{mad}^i tradeoff curves are shown that can be obtained in a similar matter, as explained in Remark 3. Observe that the MATI of the time-triggered case is slightly larger than the MIET of the event-triggered setup. The average inter-event times τ_{avg}^i, for $i \in \{1,2\}$, are obtained by simulating the system on the time interval [0, 40] for both the static and dynamic triggering conditions with $w(t)$ zero-mean Gaussian noise with variance 10 for the time-intervals $[0, 20) \cup [30, 40)$, and $w(t) = 0$ for the time interval [20, 30) and initial conditions $x_1(0) = x_2(0) = (10^{-2}, 0, 0, 0, 0, 0, 0, 0)$. The corresponding τ_{mad}^i, $i \in \{1,2\}$,

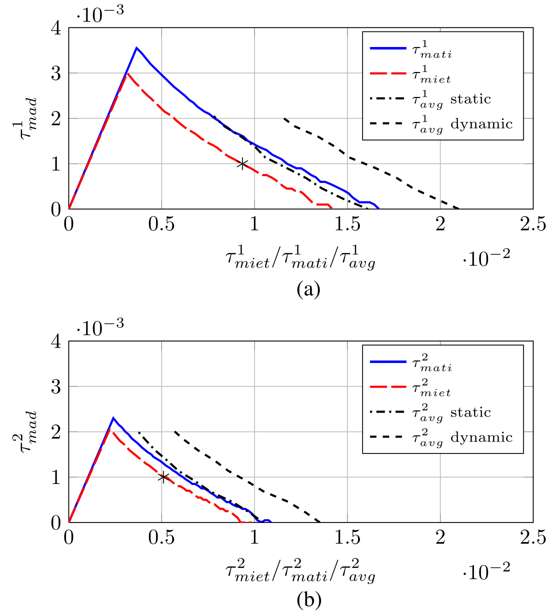

Fig. 3. Tradeoff curves of τ_{miet} versus τ_{mad} for the event-triggered setup and τ_{mati} versus τ_{mad} for the time-based specifications as discussed in Remark 3. The black curves represent the average inter-event times τ_{avg}^i obtained by means of simulations of the ETC system for various MADs, where τ_{mad}^i, $i = 1, 2$, are chosen such that $\tau_{\text{mad}}^1 = \tau_{\text{mad}}^2$. The asterisk $(*)$ in each plot corresponds to the $\tau_{\text{miet}}/\tau_{\text{mad}}$ combination that was used to produce Fig. 5. (a) Network $\mathcal{N}_1$. (b) Network $\mathcal{N}_2$.

are chosen such that $\tau_{\text{mad}}^1 = \tau_{\text{mad}}^2$. However, observe that in principle τ_{mad}^i can be chosen independently for each network $\mathcal{N}_i$, for $i \in \{1,2\}$. Based on Fig. 3, we can conclude that robust performance in terms of MADs comes at the cost of more network utilization in terms of both minimum inter-event times and average inter-event times being smaller. Furthermore, observe that the dynamic triggering condition indeed yields larger inter-event times than the static triggering condition as mentioned in Remark 5.

Fig. 4 illustrates the tradeoff between performance and network utilization in terms of an $\mathcal{L}_2$-gain and MIETs, MATIs and average inter-event times, respectively, for the case that $\tau_{\text{mad}}^1 = \tau_{\text{mad}}^2 = 10^{-3}$. The average inter-event times displayed in Fig. 4 show that for this system, better performance (in terms of a smaller $\mathcal{L}_2$-gain) comes at the cost of more network utilization. Fig. 5 shows the inter-event times of the coupled cart-pendulum systems for the case that $\tau_{\text{mad}}^1 = \tau_{\text{mad}}^2 = 10^{-3}$ and a guaranteed $\mathcal{L}_2$-gain of $\theta = 0.01$ with corresponding τ_{miet}^1 and τ_{miet}^2 as indicated by the asterisk $(*)$ in Fig. 3 for both the static and the dynamic event generators. As can be seen in this figure, the static event-triggering mechanism generates smaller inter-event times than the dynamic event-triggering mechanism, even when no disturbances are present (in this case on the time interval [20, 30]). Note that the static triggering reduces (when the system is close to the origin, e.g., in the time interval [20, 30]) to periodic triggering with the "sampling period" equal to the enforced lower bound τ_{miet} on the MIET, while the dynamic ETC does not have this property (see also Example 3 in [37]) as indicated by property (iii) mentioned in the introduction. Even though for both networks the MIET is

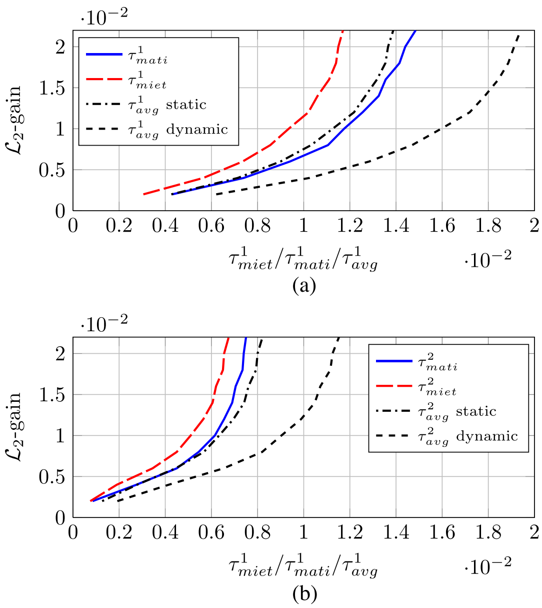

Fig. 4. Tradeoff curves of MIET, MATI and average inter-event times versus $\mathcal{L}_2$-gain for the case that $\tau^1_{\text{mad}} = \tau^2_{\text{mad}} = 10^{-3}$. (a) Network $\mathcal{N}_1$. (b) Network $\mathcal{N}_2$.

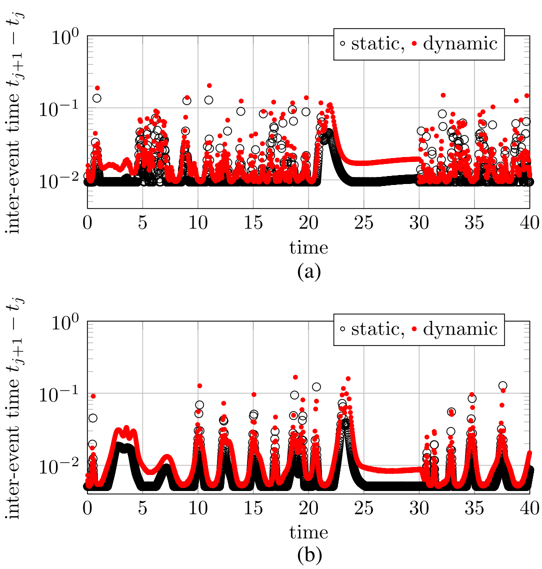

Fig. 5. Inter-event times for the static and dynamic event triggering mechanisms. (a) Network $\mathcal{N}_1$. (b) Network $\mathcal{N}_2$.

(slightly) smaller than the MATI, the *average* inter-event times generated by the dynamic ETM is significantly larger than the MATIs in contrast to the inter-event times generated by the static ETM.

B. Nonlinear Example

Consider again the control setup as shown in Fig. 2, now with the plants $\mathcal{P}_i, i \in 1,2$ given by

$$\mathcal{P}_1 : \dot{x}_1 = x_1^2 - x_1^3 + x_2 + u_1 \tag{73a}$$

$$\mathcal{P}_2 : \dot{x}_2 = x_2^2 - x_2^3 + x_1 + u_2 \tag{73b}$$

and with the controllers $\mathcal{C}_i : u_i = -2\hat{y}_i$ for $i \in \{1,2\}$, and where $\hat{y}_i$ denotes the most recently received measurement of the output $y_i = x_i$. The example is inspired by [45], but differs in the way the subsystems are interconnected. Observe that in open loop, i.e., $u_i = 0$, the system has multiple equilibria and that the origin is an unstable equilibrium point. The objective of the controller is to stabilize the origin. By defining $e_i = \hat{x}_i - x_i$ for $i = 1,2$, we have that $u_i = -2(x_i + e_i)$ and the closed-loop system can be described by

$$\dot{x}_i = x_i^2 - x_i^3 + x_{j(i)} - 2(x_i + e_i) \tag{74a}$$

$$\dot{e}_i = -x_i^2 + x_i^3 - x_{j(i)} + 2(x_i + e_i) \tag{74b}$$

where $i \in \{1,2\}$ with $j(1) = 2$ and $j(2) = 1$, as before. By taking $W_i(\kappa_i, e_i) = |e_i|$, and thus $c_i = 1$, we find by means of (74b) that

$$|\dot{e}_i| \leqslant \left|2x_i - x_i^2 + x_i^3 - x_{j(i)}\right| + 2W_i(e_i) \tag{75}$$

and thereby (51) is satisfied with $M_{e,i} = 2$ and $H_{x,i}(x) = |2x_i - x_i^2 + x_i^3 - x_{j(i)}|$ for $i \in \{1,2\}$. Consider the candidate Lyapunov function

$$V(x) = \sigma^2 \sum_{i=1}^{2} \left[\left(\alpha \frac{x_i^2}{2} + \beta \frac{x_i^4}{4}\right)\right] \tag{76}$$

where $\alpha, \beta, \sigma \in \mathbb{R}_{>0}$. By means of (74a) we find that

$$\begin{aligned}
&\langle \nabla V(x), f(x,e)\rangle \\
&= \sigma^2 \Bigg(\sum_{i=1}^{2} \big[\alpha x_i^3 - \alpha x_i^4 - 2\alpha x_i^2 - 2\alpha x_i e_i + \beta x_i^5 - \beta x_i^6 - 2\beta x_i^4 \\
&\qquad - 2\beta x_i^3 e_i\big] + 2\alpha x_1 x_2 + \beta x_1^3 x_2 + \beta x_1 x_2^3\Bigg).
\end{aligned} \tag{77}$$

Using Lemma 1 (with $\varepsilon = 1$) for the terms $2\alpha x_i e_i$, $2\beta x_i^3 e_i$ and $2\alpha x_1 x_2$ yields

$$\begin{aligned}
\langle \nabla V(x), f(x,e)\rangle \leqslant \sigma^2 \Bigg(&\sum_{i=1}^{2} \big[(\alpha^2 + \beta^2)e_i^2 + (-\beta + 1)x_i^6 + \beta x_i^5 \\
&+ (-\alpha - 2\beta)x_i^4 + \alpha x_i^3 + (-\alpha + 1)x_i^2\big] + \beta x_1^3 x_2 + \beta x_1 x_2^3\Bigg).
\end{aligned} \tag{78}$$

To find α, β, and σ such that (54) holds, we add and subtract $\varepsilon W_i^2(\kappa_i, e_i)$, $H_{x,i}^2(x)$, $\varrho(x_i)_i, i \in \{1,2\}$, and $\rho(|x|)$ with $\varrho(r) = qr^2$ for some constant $q \in \mathbb{R}_{\geqslant 0}$ and with $\rho(r) = \varepsilon r^2$ for some constant $\varepsilon \in \mathbb{R}_{>0}$

$$\begin{aligned}
\langle \nabla V(x), f(x,e)\rangle \leqslant \Bigg(&\sum_{i=1}^{2} \big[-(\varepsilon + q)x_i^2 - \varepsilon e_i^2 - H_{x,i}^2(x) \\
&+ \sigma^2(\alpha^2 + \beta^2 + \sigma^{-2}\varepsilon)e_i^2\big] + \sigma^2 p(x)\Bigg)
\end{aligned} \tag{79}$$

with

$$p(x) := \sum_{i=1}^{2} \big[x_i^2 \left(-\alpha+1+\sigma^{-2}(10+\varepsilon+q)+(\alpha-8\sigma^{-2})x_i + (-\alpha-2\beta+10\sigma^{-2})x_i^2+(\beta-4\sigma^{-2})x_i^3 + (-\beta+1+2\sigma^{-2})x_i^4\right)\big]+\beta x_1^3 x_2+\beta x_1 x_2^3 \quad (80)$$

where we used that $H_{x,i}^2(x) \leqslant 2(2x_i - x_i^2 + x_i^3)^2 + 2x_{j(i)}^2$. Note that if α, β, ε, σ, and q are chosen such that $p(x) \leqslant 0$ for all $x \in \mathbb{R}^2$, V satisfies (54) with $\gamma = \sigma\sqrt{\alpha^2+\beta^2+\sigma^{-2}\varepsilon}$. We determined numerically $[\alpha, \beta, \varepsilon, \sigma, q]$ as [3.01, 1.47, 0.01, 2.48, 0.5] to ensure that $p(x) \leqslant 0$ for all $x \in \mathbb{R}^2$ which yields $\gamma = 8.305$. By solving (29) and (30) with $\lambda_i = 0.18$, $\phi_{0,i}(0) = \lambda^{-1}$ and $\phi_{1,i}(0) = 12.9$, for $i \in \{1,2\}$, and with $L_{0,i}$, $L_{1,i}$, $\gamma_{0,i}$ and $\gamma_{1,i}$, $i \in 1,2$ according to Proposition 3, we obtain that $\tau_{\text{miet}}^i = 0.0995$ and $\tau_{\text{mad}}^i = 0.01$, $i = 1,2$. Now if we choose Ψ_i, $i \in 1,2$ according to (45), it follows from Theorem 1 that the system is UGAS under both the static and the dynamic ETM described by (6) and (47), respectively. By simulating the system 250 times on a time interval [0, 5] with the initial conditions $x_1(0)$ and $x_2(0)$ chosen randomly from the interval $[-5, 5]$, we obtained that $\tau_{\text{avg,static}}^1 = \tau_{\text{avg,static}}^2 = 0.101$ and $\tau_{\text{avg,dynamic}}^1 = \tau_{\text{avg,dynamic}}^2 = 0.137$. The minimum and average inter-event times are the same for both networks as both systems are identical. Observe that although no disturbances are present, the average inter-event time generated by the static ETM is relatively close to the MIET in contrast to average inter-event time generated by the dynamic ETM.

VIII. Conclusion

In this work, a novel *dynamic* ETC strategy for a class of nonlinear feedback systems was proposed that can simultaneously guarantee a finite $\mathcal{L}_p$-gain and a strictly positive lower bound on the inter-event times (which guarantees Zeno-freeness). In addition to these two important properties, the new ETC approach has other favorable properties: The controllers and event generators can be synthesized in an output-based and/or decentralized form with multiple asynchronously operating networks, the design takes the specific medium access protocol into account, and robustness to (variable) transmission delays is guaranteed by design. When the states of the system are close to the origin, the triggering does not reduce to periodic time-triggered control (with inter-event times equal to the enforced lower bound of the MIET). Key to obtaining all these beneficial properties is the unique combination of *dynamic* event-triggering conditions and time regularization. The design of this class of dynamic event-triggered controllers is systematic. Interestingly, the MIET and the MAD of the presented ETC strategy are close to or equal to the MATI and MAD bounds derived for time-based specifications for stability and guaranteed $\mathcal{L}_p$-gains of NCSs, but the inter-event times significantly larger. Indeed, we presented two numerical examples in which we showed that dynamic ETMs yield significantly larger average inter-event times than time-based communication and ETC strategies with static event generators, even for the case when no disturbances are present. Furthermore, we showed that the developed theory leads to tradeoffs curves between robustness (in terms of MADs), performance (in terms of an $\mathcal{L}_p$-gain of the overall system) and network utilization (in terms of MIETs and average inter-event times).

Appendix

Proof of Theorem 1: Consider the candidate Lyapunov function

$$U(\xi) = \tilde{V}(x) + \sum_{i=1}^{N} \left(\gamma_{l_i,i}\phi_{l_i,i}(\tau_i)\tilde{W}_i^2(\kappa_i, l_i, e_i, s_i) + \eta_i\right). \quad (81)$$

By means of the Comparison Lemma (see, e.g., [46, p. 102–103]), we can conclude from (7) and (32) that $\eta_i(t,j) \geqslant 0$ when $0 \leqslant \tau_i \leqslant \tau_{\text{miet}}^i$. Given this fact, observe that the triggering mechanism given by (6) and (7) ensures that $\eta_i(t,j) \geqslant 0$ for all $t \in \mathbb{R}_{\geqslant 0}$.[1] Combining this with the fact that $\phi_{l_i,i}(\tau_i) > 0$ for all $\tau_i \in \mathbb{R}_{\geqslant 0}$ and for all $i \in \bar{N}$, and the radial unboundedness of functions $\tilde{V}$ and $\tilde{W}_i$, for all $i \in \bar{N}$, due to Condition 2 and Condition 1, respectively, we can conclude that U is radially unbounded in the sense that there exist $\mathcal{K}_\infty$-functions $\underline{\beta}_U$ and $\bar{\beta}_U$ such that

$$\underline{\beta}_U\left(|\hat{\xi}|\right) \leqslant U(\xi) \leqslant \bar{\beta}_U\left(|\hat{\xi}|\right)$$

for all $\xi \in \mathbb{X}$ where $\hat{\xi} = (x, e, s, \eta) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_v} \times \mathbb{R}^{n_v} \times \mathbb{R}_{\geqslant 0}$. Hence, U constitutes a suitable candidate Lyapunov function.

Let ξ be a solution to $\mathcal{H}$ defined on the hybrid time domain $\operatorname{dom}\xi = \bigcup_{j=0}^{J-1}[t_j, t_{j+1}] \times \{j\}$ with J possibly ∞ and $t_J = \infty$ for initial condition $\xi(0,0)$ and input $w \in \mathcal{L}_p$. The function U given by (81) constitutes a valid Lyapunov function for hybrid system $\mathcal{H}$ if we can show that

$$\begin{aligned} &\langle \nabla U(\xi), F(\xi)\rangle \leqslant -\tilde{\rho}(|x|) - \delta_\eta(\eta) - \sigma_l(\tilde{W}), && \text{when } \xi \in C,\\ &U(\xi^+) - U(\xi) \leqslant 0, && \text{when } \xi \in D, \end{aligned}$$

for some positive definite functions $\tilde{\rho}$, δ_η and σ_l [40]. Note that non-strictness in the second (jump) condition is sufficient since all solutions are defined for all $t \in \mathbb{R}_{\geqslant 0}$. We can see from (15) and (28) that at transmission events, i.e., if $\xi \in D_i \wedge l_i = 0$, for some $i \in \bar{N}$ (and thus $\tau_i \geqslant \tau_{\text{miet}}^i$), we have that for all (κ_i, e_i)

$$\begin{aligned} U(\xi^+) - U(\xi) &= -\gamma_{0,i}\phi_{0,i}\left(\tau_{\text{miet}}^i\right)\tilde{W}_i^2(\kappa_i, 0, e_i, s_i)\\ &\quad + \gamma_{1,i}\phi_{1,i}(0)\tilde{W}_i^2\left(\kappa_i+1, 1, e_i, h_i(\kappa_i, e_i) - e_i\right). \end{aligned}$$

The conditions given in (22) and (29) ensure that

$$\begin{aligned} U(\xi^+) - U(\xi) \leqslant &-\gamma_{0,i}\phi_{0,i}\left(\tau_{\text{miet}}^i\right)\tilde{W}_i^2(\kappa_i, 0, e_i, s_i)\\ &+ \gamma_{1,i}\phi_{1,i}(0)\lambda_i^2\tilde{W}_i^2(\kappa_i, 0, e_i, s_i)\\ \leqslant &\,0 \end{aligned} \quad (82)$$

[1] Note that this also implies that when $\xi \in D_i \wedge l_i = 0$, the jump $\eta_i^+ = \eta_i$ is equivalent to $\eta_i^+ = 0$.

when $\xi \in D_i \wedge l_i = 0$ for some $i \in \bar{N}$. At update events, i.e., if $\xi \in D_i \wedge l_i = 1$ for some $i \in \bar{N}$, we have that due to (15)

$$U(\xi^+) - U(\xi) = -\gamma_{1,i}\phi_{1,i}(\tau_i)\tilde{W}_i^2(\kappa_i, 1, e_i, s_i) + \gamma_{0,i}\phi_{0,i}(\tau_i)\tilde{W}_i^2(\kappa_i, 0, s_i + e_i, 0). \quad (83)$$

By using the condition given by (23), we obtain that

$$U(\xi^+) - U(\xi) \leqslant -\gamma_{1,i}\phi_{1,i}(\tau_i)\tilde{W}^2(\kappa_i, 1, e_i, s_i) + \gamma_{0,i}\phi_{0,i}(\tau_i)\tilde{W}_i^2(\kappa_i, 1, e_i, s_i). \quad (84)$$

Now given the fact that, according to (19), $\xi \in D_i \wedge l_i = 1$ implies $\tau_i \leqslant \tau^i_{\text{mad}}$, the condition given in (30) ensures that when $\xi \in D_i \wedge l_i = 1$ for some $i \in \bar{N}$

$$U(\xi^+) - U(\xi) \leqslant 0.$$

With some abuse of notation, we consider the quantity $\langle \nabla U(\xi), F(\xi)\rangle$ with $F(\xi)$ given by (10), even though $\tilde{W}$ is not differentiable with respect to κ and l. Since $\dot{\kappa} = 0$ and $\dot{l} = 0$ in between jumps, this does not cause any problems. We also omit the argument of $H_{l_i,i}$. From (25), (26), and (28), we can derive that for all $\tau \in \mathbb{R}^N_{\geqslant 0}$, all $\kappa \in \mathbb{N}^N$, all $l \in \{0,1\}^N$ and almost all $(x, e) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_v}$

$$\begin{aligned}
&\langle \nabla U(\xi), F(\xi)\rangle \\
&\leqslant \sum_{i=1}^{N} \Big[-\tilde{\varrho}_i(v_i) - H^2_{l_i,i} + \gamma^2_{l_i,i}\tilde{W}_i^2 + 2\gamma_{l_i,i}\phi_{l_i,i}\tilde{W}_i \\
&\qquad \times \Big(L_{l_i,i}\tilde{W}_i + H_{l_i,i}\Big) - \sigma_{l_i,i}(\tilde{W}_i) + \Psi_i - \omega_i(\tau_i)\gamma_{l_i,i}\tilde{W}_i^2 \\
&\qquad \times \left(2L_{l_i,i}\phi_{l_i,i} + \gamma_{l_i,i}\left(\phi^2_{l_i,i} + 1\right)\right)\Big] - \tilde{\rho}\left(|x|\right) \\
&= \sum_{i=1}^{N} \Big[-M_i(\xi) - \sigma_{l_i,i}(\tilde{W}_i) + \Psi_i \Big] - \tilde{\rho}\left(|x|\right) \qquad (85)
\end{aligned}$$

with $\omega_i(\tau_i)$ as in (46), and where M_i, $i \in \bar{N}$, is given by (33). Since Ψ_i is upper bounded by $M_i(\xi) - \delta_{\eta_i}(\eta_i)$ according to (31), we obtain from (85) that

$$\langle \nabla U(\xi), F(\xi)\rangle \leqslant -\tilde{\rho}\left(|x|\right) - \sum_{i=1}^{N} \left(\delta_{\eta_i}(\eta_i) + \sigma_{l_i,i}(\tilde{W}_i)\right) \quad (86)$$

which completes the proof using standard Lyapunov arguments, see, e.g., [40], [46]. □

Proof of Theorem 2: Following the same steps as in the proof of Theorem 1, we can conclude that $U(\xi)$ is positive definite. Furthermore, we obtain that at jumps

$$U\left(\xi(t_{j+1}, j+1)\right) \leqslant U\left(\xi(t_{j+1}, j)\right) \quad (87)$$

and during flows

$$\langle \nabla U(\xi), F(\xi, w)\rangle \leqslant \mu\left(\theta^p |w|^p - |q(x, w)|^p\right). \quad (88)$$

As shown in [2], (87), and (88) imply that system $\mathcal{H}$ is $\mathcal{L}_p$-stable with an $\mathcal{L}_p$-gain less than or equal to θ which completes the proof. □

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V. S. Dolk received the M.Sc. degree (*cum laude*) in mechanical engineering from the Eindhoven University of Technology (TU/e), Eindhoven, The Netherlands, in 2013, where he is currently pursuing the Ph.D. degree in control systems.

His research interests include hybrid dynamical systems, networked control systems, intelligent transport systems, and event-triggered control.

D. P. Borgers received the M.Sc. degree (*summa cum laude*) in mechanical engineering from the Eindhoven University of Technology (TU/e), Eindhoven, The Netherlands, in 2013, where he is currently pursuing the Ph.D. degree in control systems.

His research interests include hybrid dynamical systems, networked control systems, and event-triggered control.

W. P. M. H. Heemels (F'15) received the M.Sc. degree in mathematics and the Ph.D. degree in control theory (both *summa cum laude*) from the Eindhoven University of Technology (TU/e), Eindhoven, The Netherlands, in 1995 and 1999, respectively.

From 2000 to 2004, he was with the Electrical Engineering Department, TU/e, as an Assistant Professor and from 2004 to 2006 with the Embedded Systems Institute (ESI) as a Research Fellow. Since 2006, he has been with the Department of Mechanical Engineering, TU/e, where he is currently a Full Professor with the Control Systems Technology Group. He held visiting research positions at the Swiss Federal Institute of Technology (ETH), Zurich, Switzerland (2001) and at the University of California at Santa Barbara (2008). In 2004, he was also at the Research and Development Laboratory, Océ, Venlo, The Netherlands. His current research interests include hybrid and cyber-physical systems, networked and event-triggered control systems, and constrained systems including model predictive control.

Dr. Heemels served/serves on the editorial boards of *Automatica*, *Nonlinear Analysis: Hybrid Systems*, *Annual Reviews in Control*, and the IEEE TRANSACTIONS ON AUTOMATIC CONTROL. He was a recipient of a personal VICI grant awarded by NWO (The Netherlands Organization for Scientific Research) and STW (Dutch Technology Foundation).